MODELLING GROUP LIFE RISKS AND EXPERIENCE REFUNDS

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Abstract. While experience refunds on group covers may be a good sales and client retention tool, the effect on profitability of offering such a benefit is often poorly quantified or overlooked. This paper proposes a simple mathematical framework for technically quantifying these risks on a stochastic basis.

Key-words: Group insurance, experience refund, profit share, participation, stochastic, visual basic.

1 Introduction

Group insurance is becoming a greater focus area for many insurers operating in India, and else where in Asia. Greater focus has lead to greater competition among players, and players constantly look for ways to make their product offering more attractive to potential group buyers. One such method for doing this is to offer the scheme a share in any profits that arise over a period of time.

While such offerings may be attractive to potential group schemes, the actual cost (or increase in base premium required) for offering such a scheme is often not well understood by insurers and potential clients alike.

This paper proposes a simple mathematical framework for calculating the cost of sharing profit on a stochastic basis. A description of the process is given, as well as the Visual Basic source code for a simple model.

In this paper, the phrases “profit share”, “profit refund” and “experience refund” are used interchangeably.

2 Profit share variables

In calculating profit shares, profit is usually defined as:

Profit = Premiums Earned, less
Claims Incurred, less
Expenses, less
Losses carried forward from a previous period.

For group schemes, “premium earned” and “premium written” are usually the same. Premium that is charged to the client is usually adjusted to allow for movements on and off the scheme, so that there is full correspondence between exposure and premium.

The use of “claims incurred” rather than “claims paid” would mean that some allowance in the formula would need to be made for claims “incurred but not reported" or "IBNR". However, if the profit share is calculated at a date after which final claim liability has been agreed, IBNR is ignored. If some form of IBNR is included in the claim incurred figure, then the IBNR for the previous year would also need to be considered (and removed) to make sure that the claim figure described in the "claims incurred" amount refers only to events that occurred during the period of exposure.

In this formula, expenses are usually expressed as a percentage of premiums earned, but might also include some elements based on some fixed amount, or some amount relative to sums at risk.

Whether previous years' losses are included in the profit formula is a matter for some discussion. It is usually argued that such a structure is impractical, since any scheme that suffers a loss in any one year would be inclined to move their coverage to a new insurer with a similar profit share structure in the next. In doing so, future profits in the new scheme would not be reduced by any such loss.

The profit share or experience refund is then defined as:

Profit share = Profit Refund Percentage * Profit

The profit refund percentage is usually constant, but might increase based on ratio of Profit to Premiums Earned. Alternatively, a layered approach to the profit share percentage is used, for example 50% of profit where that profit is less than 20% of earned premium, and say 75% of the portion of profit that is greater than 20% of earned premium.

3 Profit share pricing principle

The principle in pricing for profit share correctly is that the additional cost of profit share should ensure that the insurance company has the same absolute expectation of technical result as would have been the case if there was no profit share. This is done through modelling possible profit outcomes allowing for the volatility of the profit result.

4 Volatility of profit result

Volatility is created by the possibility that experience might emerge differently from how we may have expected it to. The less sure we are that experience will emerge as expected, the greater the expected volatility of result.

Given the need to price for the same expected technical result in a profit sharing environment as a non participating one, the volatility of profit that is expected to emerge is of paramount importance. The greater the volatility of expected scheme result, the greater will be the cost of the profit share. As such, the expected volatility of profit needs to be modeled in order to calculate the cost of profit share. This paper and the suggested model look only at volatility in respect of possible claim result and ignores the volatility that might be suggested in changes in inforce and resultant premium charged.

5 Claims number distributions

In a group of n identical lives with identical sums assured, with each life having the probability p of dying in any one year, the expected claims result should statistically follow a binomial distribution.

In such a case, the number of deaths that we would expect to see would be n*p.

The probability that k deaths occur in the year would be:

$$\binom{n}{k} p^k (1-p)^{(n-k)}$$

The probability that k or more deaths occur in a year would be:

$$\sum_{t=k}^{n} \binom{n}{t} p^t (1-p)^{(n-t)} \text{ , or } \quad 1-\sum_{t=0}^{k-1} \binom{n}{t} p^t (1-p)^{(n-t)}$$

As such, the variance that would be expected from such distribution would be n*p*(1-p).

Given that probability of death p is so small, there is very little difference between the mean and variance the distributed number of deaths. Therefore a common alternatively used distribution of claims outcomes that is often a lot easier to model than the binomial distribution is the Poisson distribution (whose mean and variance are the same). This leads to a simpler unbiased probability function for the number of deaths that occur in a year:

$$\frac{e^{-\text{np}} (\text{np})^k}{k!}$$

The model attached to this paper assumes that claim numbers follow a Poisson distribution.

6 Sums assured distributions

Assuming a constant probability of claim, if all lives in a group had the same sum assured, there would be no difference in the impact on profitability of one death versus another. However, usually group life schemes have variation in sum assured, with more higher salaried staff having higher sums assured (and higher ages).

Various distributions can be considered in modeling salary distributions across a portfolio of lives, but the most commonly used one is a lognormal distribution. For modeling purposes, lognormal parameters are set based on the mean and variance of the actual sums assured of a portfolio. Hence, the logarithmic ln(salary) is distributed normally with mean as follows:

$$f(\mathrm{x}) = \frac{1}{\sqrt{2\pi * \mu}} e^{-\left(\frac{(\mathrm{x}-\mu)^2}{\sigma}\right)} \quad \text{where}$$

$$\sigma = \sqrt{\ln\left(\left(\frac{\text{salary standard deviation}}{\text{salary mean}}\right)^2 + 1\right)} \quad \text{and}$$

$$\mu = \ln(\text{salary mean}) - \frac{\sigma^2}{2}$$

An alternative that is sometimes used is to set up the model to use the exact sums assured of the group in question. However, this can lead to significant increases in simulation time.

However, what is of interest is not the distribution of the possible salaries for any one life that dies, but the distribution of the sum of the possible salaries of all lives that die in any one year. This can be modeled exactly by looking at repeated lognormal distributions, or by using a method such as the Box-Müller method [1] (which is the approach adopted in the model attached to this paper).

An issue often ignored in modeling profitability of an group insurance scheme is the correlation between the claim probability and the sum assured. In reality, older lives tend to have higher sums assured. Higher mortality of older lives means that should a death occur, it is more likely that it was an older person, and hence the sum assured is more likely to be higher than average.

Without complicating the model further by modeling age distributions and resultant variation in mortality, the above issue can be taken into account by setting the best estimate claims assumption used in the model to be the sum assured weighted average expected mortality decrement. However, using an age weighted average expected mortality is the more conservative approach.

7 Factors affecting claim cost volatility

In pricing for profit share, it is not so much the actual variance or standard deviation of the claim cost distribution that affects result, it is the relationship between the standard deviation and the mean. Looking at just the claim numbers distribution, we need to look at the volatility ratio:

$$\frac{\sqrt{n * p * (1 - p)}}{n * p}, \text{ or } \sqrt{\frac{(1-p)}{n * p}}$$

Given that claim cost volatility is driven by this value, any factor that affects either the variables n or p have an impact on the claim cost volatility.

Size of scheme

As n increases the volatility ratio reduces. Hence the larger the scheme, the lower the volatility.

Age distribution of the scheme

Given that mortality increases with age, as age (and p) increases, the volatility ratio decreases. Hence an older scheme on average might have a lower cost of profit share.

Occupational classification of the scheme

As for age, heavier mortality of industrial schemes make cost of profit share cheaper than for high tech industries and other class 1 (professional class) occupation schemes.

Distribution of sums assured

Over and above the claim numbers distribution, the volatility of overall claim result is increased by increased variance in sums assured.

8 Other factors to consider in setting up a model

Confidence in best estimate mortality

Assuming a set level of best estimate mortality implies that modeling of expected profitability only looks at the systematic risk of fluctuations around that best estimate, and does not look at the risk that the best estimate assumption of itself is incorrect. If there is some level of uncertainty when setting best estimate mortality assumptions, then any profits resulting from a conservative best estimate assumption is shared with the scheme, where as losses resulting from a potentially lenient or aggressive assumption are not shared.

The most common approach for dealing with this risk is to use a slightly lighter best estimate mortality assumption in modeling the cost of profit share than was used in calculating the cost of the base premium. How much lighter would depend on the confidence in the originally placed in the assumption.

Interest

Premium is usually received at the beginning of a scheme year, with claims being paid out just after mid year on average, and profit share returned at the end of the year. When equating expected margin in premium between profit sharing structures and non-participating structures, the timing of cashflows also needs to be taken into account.

Changes in demographics

Of course all of the claim cost assumptions are based on the specific demographics of the scheme. Consideration of how demographics might change over time should be taken in setting modelling assumptions.

9 The profit simulation

A model is created to calculate the expected result of a group scheme based on the above distributions. Each simulation would calculate the profit margin arising from a scheme as:

Profit = Premiums Received – Expenses – Sum of Claims (Number of claims)

Sum of Claims (Number of claims) would be the simulated amount based on the above discussed distributions. Firstly, a "Number of Claims" is simulated based on the Poisson distribution described, and from that number, a "Sum of Claims" is calculated based on the sum of lognormals.

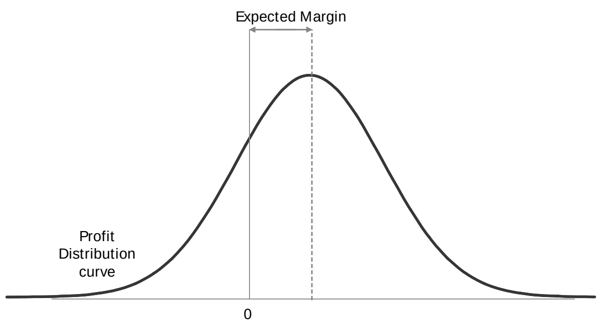
Premiums Received and Expenses would be fixed for each simulation.

10 The profit distribution and refund calculation process

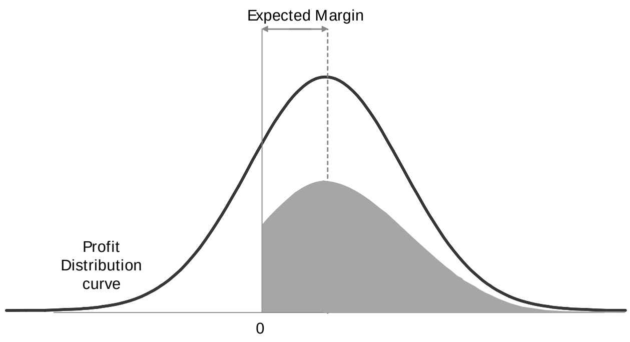
Repeated simulations of this profit result gives a distribution of profits around an expected mean. For each of these scenarios, a calculation is made of the profit share refund that would apply. This profit share refund amount is then added and averaged over all the simulations and then compared with the original premium amount to determine what profit share loading should have applied in order to fund that profit refund. This profit share loading is then applied to the original premium, and the whole simulation repeated on this new premium to calculate the new profit share refund proportion. This process is repeated again until the increases in premium required in successive iterations reaches a targeted minimum.

The above short description can be explained in more detail and broken down pictorially as shown below:

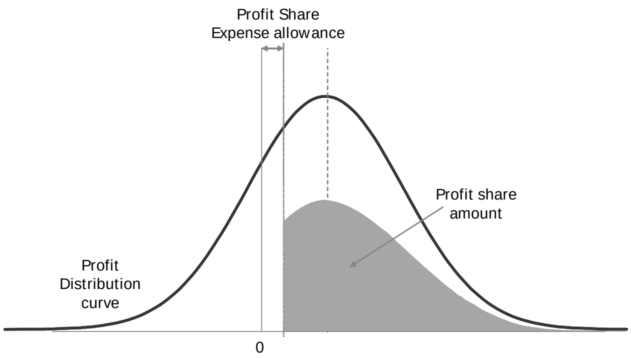
1. As the number of simulations increases, the distribution of possible profits/results tends towards normal, with the mean being the expected level of margin over the expected claim cost and expenses.



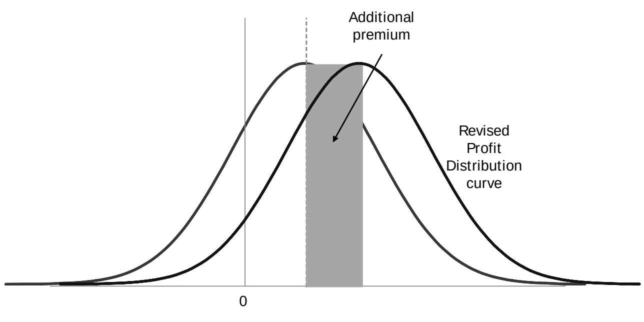
2. The area under the curve represents the overall profitability of the scheme. Where a profit on the scheme is expected, an amount of profit refund needs to be returned to the scheme. For example, if 60% of total profit is to be refunded, this is represented by the mustard area shown on the graph below.



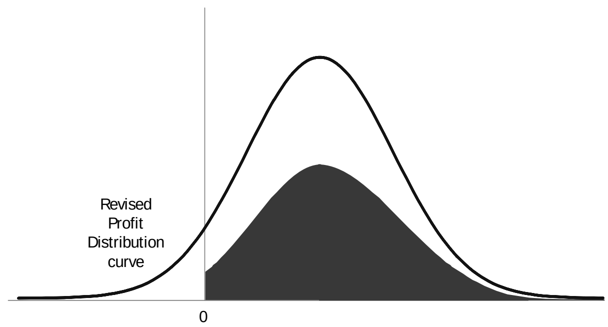
3. From this profit amount, the expense allowance from the profit share formula must be removed. The shaded area now represents the cost of the profit share for the given simulation.



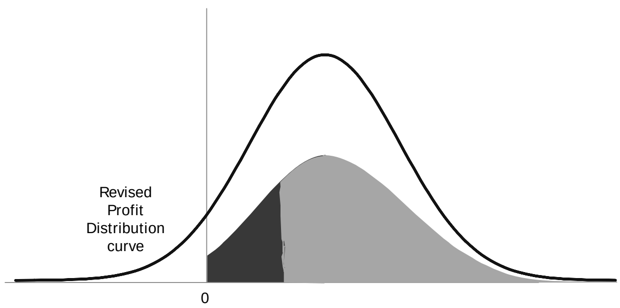
4. If this cost of profit share area is reshaped into a rectangle cost of profit share is added to the premium and re-simulated, a different set of possible profitability outcomes is generated.



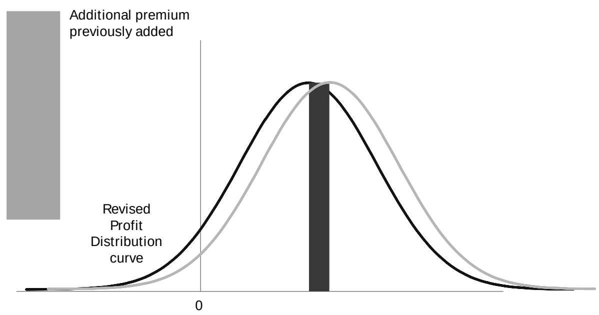
5. This in turn creates a different (larger) cost of profit share, since given the larger premium, a larger proportion of that premium will be returned as profit.



6. However, a large portion of this cost of profit share has already been charged for and so it is only the remaining amount that must be added, and the process re-iterated.



7. This process continues until a the consecutive differences in resultant additional premium required become insignificant.



11 An Excel model interface

A simple Excel model can be created to perform the simulations required. Such a model would use Visual basic to run the simulation based on inputs from the Excel model. A sample inputs screen might look as follows:

Simple Group Profit share calculator

Portfolio Parameters

Reinsured Portfolio Size	2,000
Average Sum Insured	200,000
Std Dev Sum Insured	200,000

Profit Share Formula a(eP-C)

Percentage Profit (a)	50%
Percentage Expenses (e)	90%

Calculated WP Loading	21.0%

Basic Premium parameters

Best Estimate Claim Rate	2.000	per mille
Per Mille expense	0.20	
Net Premium Loading	5.00%	
Gross Premium Loading	7.00%	
Final Gross Premium	2.992	

Simulation parameters

Number of Simulations	40,000
Interest	5.0%

Calculate loading

12 Areas where the model can be improved

Uncertainty surrounding the best estimate claim rate

As it stands, the model currently assumes that the best estimate claim rate is the true mean of the expected group mortality rate. However this is unlikely to be the case in reality. Often, we are not entirely sure of what the best estimate claim rate should be. If we were to overstate our best estimate mortality assumption, we are at risk of underpricing the cost of profit share.

In order to take this into account, it is possible to fit a distribution to the best estimate claim rate used in the stochastic model, according to the level of comfort that we have in the figure. Adding volatility to this parameter would ultimately have the effect of increasing the cost of experience refund.

The model as it stands can be used for deterministic scenarios of best estimate claim rate in order to get a feeling for the sensitivity of the profit share cost to the best estimate claim rate assumption. The degree of conservatism in setting the best estimate claim rate assumption could be set depending on the distribution of likely outcomes.

Loss of experience refund on non-renewal

Many insurers require that group schemes renew their insurance in order to participate in previous years profit. In this way, experience refund is sometimes provided as a discount to renewal premium rather than a profit share.

As such, it could be argued that the cost of profit share should be modeled over a number of years rather than just 1 year. In a multi year model, the expected profitability of the scheme in a non-participating environment needs to be adjusted by the probability that the scheme might not renew in future years, as compared with the participating environment where renewal after a profitable year is almost guaranteed. Hence, the model would equate technical result over a number of years taking into account the probability that a scheme might not renew in a non-participating environment.

Correlation between age and sum assured

As mentioned earlier, the model assumes a single expected decrement rate, and doesn't allow for the fact that older lives (that are more likely to die) tend to have a greater average death benefit. The best way to take this into account would be to model the actual lives data from the scheme being priced in order to sample likely claims cost outcomes. However, this would add considerably to simulation time and complexity, and might give spurious results given the difficulty in accurately setting the mortality assumptions correctly.

Changes in scheme demographics

The model currently assumes that the demographics of the scheme remain constant throughout the scheme year. In reality, everything from average sums assured to expected claim decrements would change as some lives leave and new lives join the scheme. Attempts at modelling these would probably lead to spurious levels of accuracy, but cognisance of the likely changes that might occur in demographics could be taken in setting initial assumptions.

13 Conclusion

Calculating the exact cost of a profit share option in pricing a group scheme can be done on a technical basis. A sample set of visual basic code that could be used for calculating profit share based on the above description is included in this paper as an appendix.

An Excel model for use in pricing is available from the author on request.

References

[1] Box, G. E. P. and Muller, M. E. (1958), A Note on the Generation of Random Normal Deviates, Ann. Math. Stat. **29**, 610-611

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Appendix

Sample source code – Visual basic for Excel

```
'Simple Group Profit Share Pricing Model
'Written by Walter de Oude
'Prepared for presentation at 13th EAAC, Bali 12-15 September 2005

Option Explicit

Public PortSize, FinalPremRate, ClaimRate As Double
Public NetPremMargins, PerMillePremLoad, GrossPremMargins As Double
Public ProfitShareLoading As Double
Public NonProfitPremRate As Double
Public AveSumInsured As Double
Public StDevSumInsured As Double
Public LogMu As Double
Public LogSigma As Double
Public PSPercProf As Double
Public PSPercExp As Double
Public i, j, Simulations As Long
Public Interest As Double
Public ExpectedNumberOfClaims As Double
Public NumberOfClaims() As Integer
Public ClaimsCost() As Double
Public Premium As Double
Public ProfitShare() As Double
Public Profit() As Double
Public AverageNumberOfClaims() As Double
Public AverageClaimsCost() As Double
Public TotalPVExpectedProfit As Double
Public AveragePVProfit As Double
Public Const Pi = 3.14159265358979

Sub CalculatePremium()
  NonProfitPremRate = (ClaimRate * (1 + NetPremMargins) + PerMillePremLoad) / _
          (1 - GrossPremMargins)
  FinalPremRate = NonProfitPremRate * (1 + ProfitShareLoading)
  TotalPVExpectedProfit = PortSize * (NonProfitPremRate * (1 - GrossPremMargins) + _
         FinalPremRate * GrossPremMargins - ClaimRate * (1 + Interest) ^ (-0.5)) * _
         AveSumInsured
End Sub

Sub Participation()
   InitialiseVariables
   Rnd (-1)
  ' Rnd(-1) is done to make sure that the same result is generated
  ' for the same set of parameter inputs every time
   CalculatePremium
   AveragePVProfit = 1
   While Abs(TotalPVExpectedProfit / AveragePVProfit - 1) > 0.003
  ' We repeat the process until the result comes close to an equilibrium
   SimulateClaimNumbers
  ' to get a number of claims
   SimulateClaimsCost
  ' for the number of claims, what is the resultant cost
   CalculateProfitShare
   CalculatePremium
   Wend
End Sub
```

```
Sub InitialiseVariables()
' This procedure takes inputs required as tabulated in the Excel spreadsheet
    PortSize = Range("PortSize").Value  'The size of the portfolio
    ClaimRate = Range("ClaimRate").Value / 1000   'The rate of mortality decrement
    AveSumInsured = Range("AveSumInsured").Value   ' Average sum insured of the portfolio
    StDevSumInsured = Range("StDevSumInsured").Value
    NetPremMargins = Range("NetPremMargins").Value   'Loading applicable to claim rate
    PerMillePremLoad = Range("PerMillePremLoad").Value / 1000 'e.g. Capital charges
    GrossPremMargins = Range("GrossPremMargins").Value 'e.g. commissions
    PSPercProf = Range("PSPercProf").Value 'The percentage of profit refunded
    PSPercExp = Range("PSPercExp").Value  'The percentage of premium used in profit calc
    Simulations = Range("Simulations").Value ' Number of simulations
    Interest = Range("Interest").Value
    ProfitShareLoading = 0
    LogSigma = (Log((StDevSumInsured / AveSumInsured) ^ 2 + 1)) ^ 0.5
    LogMu = Log(AveSumInsured) - 0.5 * LogSigma ^ 2
End Sub

Sub SimulateClaimNumbers()
    ExpectedNumberOfClaims = PortSize * ClaimRate
    ReDim NumberOfClaims(Simulations) As Integer
    For j = 1 To Simulations
        NumberOfClaims(j) = Poisson(ExpectedNumberOfClaims)
    Next j
End Sub

Sub SimulateClaimsCost()
    ReDim ClaimsCost(Simulations) As Double
    For j = 1 To Simulations
        ClaimsCost(j) = SumLogNormal(NumberOfClaims(j))
    Next j
End Sub

Sub CalculateProfitShare()
    ReDim ProfitShare(Simulations) As Double
    ReDim Profit(Simulations) As Double
    Dim ProfitAccount As Double
    AveragePVProfit = 0
    Premium = PortSize * FinalPremRate * AveSumInsured
        For j = 1 To Simulations
            ProfitAccount = PSPercExp * Premium - ClaimsCost(j)
            If ProfitAccount > 0 Then
                ProfitShare(j) = PSPercProf * ProfitAccount
            Else
                ProfitShare(j) = 0
            End If
            Profit(j) = Premium - ClaimsCost(j) * (1 + Interest) ^ (-0.5) - _
                        ProfitShare(j) * (1 + Interest) ^ (-1)
            AveragePVProfit = AveragePVProfit + Profit(j)
        Next j
    AveragePVProfit = AveragePVProfit / Simulations
    Premium = Premium + TotalPVExpectedProfit - AveragePVProfit
    ProfitShareLoading = Premium / (NonProfitPremRate * PortSize * AveSumInsured) - 1
    Range("PSLoading").Value = ProfitShareLoading
End Sub

Function Poisson(L As Double) As Integer
' Generate Random Number from a Poisson Distribution
    Dim PoiTarg As Double
    Dim PoiRes As Double
    Dim PoiCount As Integer
    PoiTarg = Exp(-L)
    PoiCount = 0
```

```
    PoiRes = Rnd
    Do While PoiRes > PoiTarg
        PoiRes = PoiRes * Rnd
        PoiCount = PoiCount + 1
    Loop
    Poisson = PoiCount
End Function

Function SumLogNormal(N As Integer) As Double
' Generate Sum of N LogNormal(LogMu, LogSigma) Random Variables
' Box-Müller Method
    Dim X1, X2 As Double
    Dim R1, R2 As Double
    Dim LNRes As Double
    LNRes = 0
    If N = 0 Then
        LNRes = 0
    ' Even Number of Claims
    ElseIf N Mod 2 = 0 Then
        For i = 1 To N / 2
            R1 = Rnd
            If R1 = 0 Then
                R1 = 1E-16
            End If
            R2 = Rnd
            X1 = (-2 * Log(R1)) ^ 0.5 * Cos(2 * Pi * R2)
            X1 = Exp(X1 * LogSigma + LogMu)
            X2 = (-2 * Log(R1)) ^ 0.5 * Sin(2 * Pi * R2)
            X2 = Exp(X2 * LogSigma + LogMu)
            LNRes = LNRes + X1 + X2
        Next i
    ' Odd Number of Claims
    Else
        For i = 1 To (N - 1) / 2
            R1 = Rnd
            If R1 = 0 Then
                R1 = 1E-16
            End If
            R2 = Rnd
            X1 = (-2 * Log(R1)) ^ 0.5 * Cos(2 * Pi * R2)
            X1 = Exp(X1 * LogSigma + LogMu)
            X2 = (-2 * Log(R1)) ^ 0.5 * Sin(2 * Pi * R2)
            X2 = Exp(X2 * LogSigma + LogMu)
            LNRes = LNRes + X1 + X2
        Next i
        R1 = Rnd
        If R1 = 0 Then
            R1 = 1E-16
        End If
        R2 = Rnd
        X1 = (-2 * Log(R1)) ^ 0.5 * Cos(2 * Pi * R2)
        X1 = Exp(X1 * LogSigma + LogMu)
        LNRes = LNRes + X1
    End If
    SumLogNormal = LNRes
End Function
```