SOME REFLECTIONS ON ACTUARIAL RISKS AND THEIR MODELLING

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Abstract. An actuary has to deal with several sources and types of risks. This is fundamental to his profession. There is a sort of tension between the responsibility of the actuary on the one hand and financial management and marketing on the other. To the actuary reserve, solvency, and resilience are important. The shareholders are especially interested in profit and the value of the insurance company or pension fund. Therefore, management organizes embedded value and fair value calculations on a regular base. Under pressure of Basel II to strive for beter financial stability it has become unavoidable to apply stochastic models. For several sources of risks adequate techniques based on stochastic models are required. As it was stated in the theme of the conference it is not sufficient for the profession to respond through stronger common education requirements. Specialization and permanent education have to be strongly stimulated.
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1. Introduction

The theme of the 13th East Asian Actuarial Conference is: "the actuary at risk". An actuary is a professional dealing with several sources and types of risks. By education, training and experience an actuary is very well equipped to calculate premiums, to evaluate the risks of policies and portfolios, and to judge whether the reserves are prudential enough. These competences have a long lasting and impressive history. To the actuary reserve, solvency, and resilience are important concepts.

An insurance company has several stakeholders, i.e. the shareholders, the policyholders and the employees. One may think that the actuary especially is a kind of defender of the rights of the policy holders and as consequence aims at continuation of the activities of the insurance company or pension fund on the long term. This will be in line more or less with the aims of the regulator. On the other hand, it is clear that the shareholders want to be rewarded for their investments. To stay in business it is very important for an insurance company to realize sufficient returns from investments, to continuously reduce costs at all levels of the company, to conclude a sufficient number of policies, and to reinsure high losses. This has to be realized in competition with other insurance companies. In order to compete quality, marketing and innovation are relevant issues. To keep the customers satisfied quality of the service is important. To attract new customers or to keep the policyholders new products have to be developed that successfully can be sold in the market. Here we see activities of financial managers, marketeers and acturies. On a global company level the actuary will be involved in embedded value or fair value calculations. A modern actuary has acquired knowledge on investments and certain fancy financial derivatives. He is able to cooperate with financial managers and marketeers. This requires a multidisciplinary attitude and good communicative skills.

Originally, an important task of the actuary was to take the necessary precautions in order to ensure that the policyholders receive the benefits they have a right to and to ascertain that the right premiums are set. The actuarial calculations, however, have a kind of build-in mechanism of dealing with risks in a conservative manner. In some companies, as they told me, they make profit because of the conservative estimation of the costs incorporated in the gross premium. The attitude that the actuary calculates the (expense-loaded) premiums, certifies the necessary reserves and leaves the other issues in running an insurance company to financial managers leads to isolation of the actuary, because it is considered too easy to claim which reserve has to be available and to lean back and watch what happens. Financial managers are sometimes suspicious about the necessity of the height of the benefit reserves as required by the actuary. Because of competiton there is a pressure on the height of the premiums and the kind of promises towards the policyholders. Moreover, there is a tendency that insurers make explicit what the net premiums and the costs are. Nowadays the interest rate is low. Therefore good returns from investments are needed in order to be able to keep the promises with regard to e.g. pensions. We know that this is risky if we look at the developments on e.g. the stock exchanges.

In the Netherlands, pension funds had considerable losses. This has led to more severe rules by the regulator. The modern actuary will have to accept joint responsibility together with the financial managers for the complete performance of the insurance company. This means that actuaries become involved in assessing risks that are of a financial nature. From a sociological point of view, if the actuary wants to keep his highly-valued position, he will have to adapt to new circumstances and should be willing to take more responsibilities. For instance, actuaries become involved in ALM studies and in risk management. In this position the actuary will really be at risk. Actuaries have to specialize and this will lead to specialization trajectories in the educational programs and to permanent education of professionals.

The role of the actuary also changes because of the merging of banking and insurance activities. ING is an example of a successful merger. This kind of cooperation of bankers and insurers has to do with the use of each others distribution channels. Moreover, nowadays various kinds of fancy products are engineered that are combinations of a financial part and an insurance part. An example is a mortgage in combination with an endowment. Another simple example will be discussed in more detail in section 2.

Much progress is made in developing valuation techniques for complicated products. It will take time before the findings are incorporated in the common toolbox of actuaries. Indeed, "the ultimate guarantee for the world of consumers of actuarial services are uniformly high quality standards of practice that will need to be supported by the wider use of state-of-the-art techniques and models that are applicable throughout the actuarial universe" (theme EAAC13). We cannot stop learning and therefore permanent education is of utmost importance.

On its website (`www.soa.org`) we read that "The Society of Actuaries is a nonprofit educational, research and professional society of 18,000 members involved in the modelling and management of financial risk and contingent events." Valuation was and will be the most important task of an actuary, but new products require new techniques. As an example, see section 2, one of my students, mr. Edward Christian, studied the problem of valuation of a term insurance and a pure endowment in his master's thesis. There was a minimum guaranteed benefit, but the ultimate benefit also depends on the value of a risky asset at the moment the benefit has to be paid. This is a mixture between an insurance product and a financial product. In order to derive the net single premium of the benefit one has to make assumptions about the behaviour of the price of this risky asset. A more or less classical assumption is to model the behaviour by geometric Browninan motion. This leads to formulas that are used in European option pricing. This illustrates that acturies may benefit from results that are well known in financial engineering.

In section 2 we adopt a fixed technical interest rate. Section 3 gives a brief account to the situation that one has to deal with fluctuating interest rates. Section 4 gives some general remarks on stochastic models and section 5 presents the conclusions.

2. A simple combined actuarial and financial model

For an introduction to stochastic investment modelling and investment risk management we refer to chapters 1-5 of [1].

Valuation was and will be the most important task of an actuary, but new products require new techniques. As an example, one of my students, mr. Edward Christian from Institut Teknologi Bandung, Indonesia and the University of Groningen, the Netherlands studied the problem of valuation of a term insurance and a pure endowment in his master thesis [3]. The contents of this section will be particularly based on his thesis. There was a minimum guaranteed benefit, but the ultimate benefit also depends on the value of a risky asset at the moment the benefit has to be paid. This is a mixture of an insurance product and a financial product.

For an age (x) let T_x be his remaining lifetime. We consider a term insurance and an endowment. In the classical term insurance model, we would consider a fixed benefit B to be paid at the moment of death in case of death before time n denoting the duration of the policy. In case of a pure endowment, a benefit B will be paid if the insured is alive at time n. The net single premiums are $B\overline{A}^{1}_{x:\overline{n}|}$ and respectively $B\overline{A}_{x:\overset{1}{\overline{n}|}}$. A more complicated policy occurs if the benefit at time time t is equal to the value P_t of a risky asset at that time, but it is guaranteed that the benefit is least B. In case of the term insurance the benefit will be $\max(P_t, B) = B + (P_t - B)^+$ in case of death at time $t \in [0, n]$. For the pure endowment the benefit is equal to $B + (P_n - B)^+$ at time n in case of survival. In order to obtain the net single premiums we have to calculate

$$B\overline{A}^{1}_{x:\overline{n}|} + \mathrm{E}\exp(-\delta T_x)(S_{T_x} - B)^+ I_{[0,n]}(T_x)$$

respectively

$$B\overline{A}_{x:\overset{1}{\overline{n}|}} + {}_np_x \exp(-\delta n)\mathrm{E}(S_n - B)^+.$$

If we look at $\mathrm{E}\exp(-\delta t)(S_t - B)^+$, then we recognize the present value of a so-called European call option. If P_t follows a geometric Brownian motion, i.e. $P_t = P_0 \exp((\mu - \sigma^2/2)t + \sigma W_t)$, where μ is the mean rate of log-return, σ measures the constant volatility of the asset and W_t is standard Brownian motion, then according to the classical Black-Scholes-Merton formula

$$\mathrm{E}\exp(-\delta t)\mathrm{E}(S_t - B)^+ = P_0\Phi(c) - Be^{-\delta t}\Phi(c - \sigma\sqrt{t}),$$

where Φ denotes the distribution function of the standard normal distribution and

$$c = \frac{\log(P_0/B) + (\delta + \sigma^2/2)t}{\sigma\sqrt{t}}.$$

This illustrates that acturies may benefit from results that are well known in financial engineering. In order to derive the net single premium of the benefit one has to make assumptions about the behaviour of the price of this risky asset such as geometric Browninan motion. It is also assumed that the volatility is fixed. This leads to formulas that are used in European option pricing: the Black-Scholes-Merton formula. One of the questions that should be asked is how well does this model fit to the historical price development in the past. It may appear that the assumption of geometric Brownian motion is not a good one, because the logarithms of the prices measured do not follow a normal distribution. One may indeed expect that the true probabilty distribution wil have heavier tails. If that is the case then we have to look for other stochastic processes than geometric Brownian motion that will better fit to the obeserved measurements. This leads to the consideration of so-called Lévy processes and to the use of infinitely divisible distributions. In [3] this aproach was followed and the so-called Meixner distribution was applied. The usual machinery of actuarial calculations cannot be used anymore. Advanced computational techniques have to be applied to arrive at the necessary answers. Statistical skills are needed to judge the empirical foundation of the stochastic models. Moreover, for the necessary calculations knowledge of numerical techniques are indispensable. To be a little bit more precise, multiple integrals have to be evaluated. The convergence of numerical algorithms can be slow and modern Monte Carlo simulation techniques can often successfully be applied. It is already difficult to analyse one policy whose value is partly based on a risky asset. On a technical level, how does the total claim amount of a portfolio behave? A complicating factor in this respect is that usually net present values of the benefits in a portfolio are considered to be independently distributed. This is a reasonable assumption for fixed technical interest rates in a life insurance portfolio. It may also be reasonable for e.g. a health insurance portfolio, but it becomes questionable in case of an insurance for earthquakes. The benefits in the portfolio of our combined financial product are correlated just because of the underlying price process of the risky asset.

3. Fluctuating interest rates

The assumption that an interest rate is fluctuating will frequently lead to the assumption that it is a random variable. For an introduction we refer to chapter 21 of [2]. The deviation of the interest rate from the fixed technical interest rate leads to a so-called technical gain. This is discussed in chapter 6 of [4].

A recent article on the premium and the risk of a life policy in the presence of interest rate fluctuations is [7]. In the previous section we questioned whether the assumption of Brownian motion is adequate. As it was formulated in [7], "the model adopted to describe the interest rate uncertainty, in a continuous framework, has usually involved the use of a Brownian motion". In [7] the general idea of [6] is followed. We are interested in the instantaneous interest rate process (or, force of interest process) $\{\delta_t; t \geq 0\}$. One way of modelling could be to model this process directly. Well-known models are:

- the Vasicek model $d\delta_t = a(b - \delta_t)dt + \sigma dW_t$,
- the Cox-Ingersoll-Ross model $d\delta_t = a(b - \delta_t)dt + \sigma\sqrt{\delta_t}dW_t$.

Alternatively, one may start from the discount factor $\Delta_t = \exp(-\int_0^t \delta_s \, ds)$. The model is then of the form $\Delta_t = \theta t + X_t$, where

- X_t is a Brownian motion,
- X_t is a reflected Brownian motion, or
- X_t is an Ornstein-Uhlenbeck process.

For references and elaborations we refer to [6] and [7]. We also refer to section 24.6 of [1].

This case illustrates the importance of stochastic models. In life insurance we use a kind of fixed technical interest rate that was considered to be conservative, but nowadays the real interest rate is low and sometimes even lower than the technical interest rate. This can only be the case if there are relatively high returns on the premiums collected and the costs are conservatively estimated. However, there is a tendency that insurance companies explicitly announce the costs they bring into account. In the insurance market there will be competition on costs. To be viable as an insurer good returns om investments on the long term are very important.

Another approach is followed in e.g. [5]. This is research jointly with Mrs. Lienda Noviyanti from the Department of Mathematics, Institut Teknologi Bandung, Indonesia, which was ininitated when she visited the Department of Econometrics, University of Groningen, the Netherlands. The idea is in line with [4], section 5.7. Stochastic independence of policies is lost in the previously mentioned models, because all policies depend on the same stochastic interest rate. In actual practice, an actuary may require that the usual acutarial tools can be applied. Therefore, e.g. the net present value of actuarial products with varying interest rates have to be bounded as accurate as possible by a lower and an upper bound in terms of a present value of a classical actuarial quantity with a kind of average technical interest rate. For example, the present value of a continuous benefit payment is

$$\overline{A}^*_{\overline{T|}} = \exp(-\int_0^T \delta_s \, ds).$$

In a similar way one could write

$$\overline{A}^*_{\overline{T|}} = \exp(-T\delta^*_T),$$

where

$$\delta^*_T = \frac{1}{T}\int_0^T \delta_s \, ds$$

denotes the mean of the function δ_s over the interval $[0, T]$. If one assumes that $\delta^*_T \to \delta^*_\infty$ one may use bounds such as

$$c_l \overline{A}_{\overline{T|}} \leq \overline{A}^*_{\overline{T|}} \leq c_u \overline{A}_{\overline{T|}},$$

where

$$\overline{A}_{\overline{T|}} = \exp(-T\delta^*_\infty).$$

It is also possible that one wishes to bound $\delta_l \leq \delta^*_T \leq \delta_u$ and apply bounds

$$\overline{A}_{\overline{T|}}(\delta_l) \leq \overline{A}^*_{\overline{T|}} \leq \overline{A}_{\overline{T|}}(\delta_u).$$

4. Stochastic models

In sections 3 and 4 we briefly indicated some ways to deal with randomness. The use of stochastic models will increase because of Basel II. The idea is that insurance companies have to give more insight in the uncertainties concerning reserves, solvency, etc. and to express the risks in a more quantitative way.

In the last three decades one may observe that the way of thinking and reasoning in actuarial science and practice is drastically changing from a deterministic (sometimes algorithmic) approach towards the implementation of stochastic models. Deterministic methods also incorporate the use of margins which may be implicite or explicite in order to protect against variability. The disadvantage of these methods is that it is intrinsically not possible to construct statements about the size of the risk that such margins are exceeded. However, if such a rule is applied in practice, then in course of time empirical evidence can be collected as to the relative frequency of the event of exceedence. The risk can therefore only be "measured" after a reasonable number of "experimemts" in real actuarial practice. Every actuary will agree that it is not professional to estimate risks on the basis of experiments in going concern of the insurance company. The advantage of stochastic models is that such models in principle can be used to calculate certain risks. Obviously, it depends on how well the stochastic model describes the acturial context at hand, how accurate the risk estimate will be. At least stochastic models can provide insight in the possible outcomes.

Experience with macro-economic forecasting learned the author that a macro-econometric model has to beat the no-change forcasts on the short-term and that this is really not so simple. Long-term forecasting cannot be done without macro-econometric models. So, for the valuation of actuarial products, it is important to make a distinction between products that expire on the short term and products that expire on the long term. In the first case the value will be estimated on the basis of relevant statistical data from the past by means of e.g. generalized linear models, etc. In the second case, the choice of a suitable stochastic model becomes much more important.

5. Conclusions

The discipline of actuarial science develops strongly. By means of a couple of technical examples it is illustrated what kind of techniques are involved nowadays. Modern actuaries will have to operate in a strongly multidisciplinary environment. Actuaries wil specialize to keep up with recent developments. They cooperate with financial engineers, marketeers and financial risk managers. There wil be an increasing pressure of regulators to make use of stochastic models to evaluate the possible risks. Moreover, in case of embedded value calculations or ALM studies quite a lot of stochastic models will be needed to model the relevant aspects in order to make the necessary forecasts and/or extrapolations. In the near future we will face questions like "how to deal with risks that are technically hard to calculate" and "what will be a satisfactory system of dealing with certain common valuation problems".

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