STOCHASTIC MODELS FOR PREMIUM CALCULATION UNDER SYARIAH LAW

Ocke Kurniandi

PT. MAA Life Assurance, Indonesia

Abstract. This paper introduces the alternative method to calculate premium that is compliance under syariah law. There is distinction between conventional method and syariah principles about interest rate. The method is based on stochastic model and uses stochastic differential equation. It's easier use numerical approach, than analytic approach. As case study we examine this method to five-year life insurance coverage by Monte-Carlo simulation.

Key-words: syariah principles, stochastic model, stochastic differential equation numerical approach, Monte-Carlo simulation.

1 Introduction

Many life insurances in Indonesia have been selling the products under syariah law. Principles of syariah law for insurance are the insurance product that must not contained ghoror (unexplainable rule), maisir (gambling), and riba (interest rate or guaranteed investment returns) (Dewan Syariah Nasional Majelis Ulama Indonesia, 2003). In practice the actuaries always assume the interest rate when calculating the premium. If we assume interest rate or give the guaranteed investment return then the premium calculation can't be accepted under syariah law. Now, we have distinction between conventional method and syariah principles.

Many books in actuarial science put interest rate as a deterministic variable but now some books introduce interest rate as a random variable. If the interest rate as a random variable than this assumption is matched with syariah principles. However, that we start to think the investment return or interest rate as a random variables. Later we can find many different formulations and perception between conventional and syariah product. We start to find the premium calculation formula as first step to formulating the syariah actuarial method.

Basic economic theory would suggest that interest rate, like prices, are established by supply and demand. If the demand for funds to borrow is strong in relation to the availability of funds, interest rates will rise. Conversely, if the demand for funds to borrow is weak in relation to be availability of funds, interest rates will fall. In syariah principles, the theory's a little bit different because they use profit sharing concept then borrow and lender have same position on investment return. The major factors which have an influence on level of investment return

1. Productivity growth in economy
2. Inflation
3. Risk and uncertainty
4. Governmental policy
5. Random fluctuation

In stochastic models the major factors will be expressive as drift and volatility factor. The drift and volatility factors are measurement of economy and governmental condition.

2 Stochastic Process

Stochastic process used in stock market and the model doesn't give a guaranteed interest rate. In many cases stochastic process have been used to predict the investment return or interest rate and this method assume the interest rate or investment return is a random variable (Kellison, 1991; Baxter, 1996)

Definition 2.1 *Stochastic Process* $\{ r_t , t \geq 0 \}$ *as Browns motion with drift* μ *and variance* σ *if*

1. $r_0 = 0$
2. $\{ r_t , t \geq 0 \}$ *stationer and independent increment*
3. r_t *normal distribution with* $\mu\, t$ *and variance* $\sigma\, t$

Generally, one assumes that the value of random variable of investment return will grow with time. Let us consider the difference equation

$$\Delta r = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t} \tag{1}$$

Here μ (the drift) is a measure of growth rate and σ (the volatility) is a measure of the variability of the process as time increases. ε is normal distribution with mean zero and variance 1. In the limit, as Δt goes to zero, such a process is uniquely defined and commonly refereed to as a Brownian process.

$$dr = \mu\, dt + \sigma\, dw \tag{2}$$

Let μ and σ are function with two variable, *t* and *r*.

$$dr = \mu(r,t)dt + \sigma(r,t)dw \tag{3}$$

The equation (3) is called a stochastic differential equation (SDE) for *r* (Baxter, 1996; Rolski, 1999; Fabozzi 2002) As in ordinary differential equation (ODE), an SDE need not have solution, and if it does it might not be unique.

We model the investment return as

$$dr = (\theta - \alpha\, r_t)dt + \sigma\, dw \tag{4}$$

for some constant θ, α and σ. The SDE is composed of Brownian part and restoring drift which pushes it upwards when the process is below θ / α and downward when it is above. The magnitude of the drift is also proportional to the distance away from this mean.

We can use Ito's formula to check that the solution to this, starting *r* at r_0, is

$$r_t = \frac{\theta}{\alpha} + e^{-\alpha t}\left(r_0 - \frac{\theta}{\alpha}\right) + \sigma e^{-\alpha t}\int_0^t e^{\alpha s} dw_s \tag{5}$$

As it happens r_t can be rewritten in terms of a different Brownian motion W as

$$r_t = \frac{\theta}{\alpha} + e^{-\alpha t}\left(r_0 - \frac{\theta}{\alpha}\right) + \sigma e^{-\alpha t} W\left(\frac{e^{2\alpha t} - 1}{2\alpha}\right) \tag{6}$$

so that r_t has a normal marginal distribution with mean $\frac{\theta}{\alpha} + e^{-t}\left(r_0 - \frac{\theta}{\alpha}\right)$ and variance $\sigma^2\left(1 - e^{-2\alpha t}\right)/2\alpha$. As t gets large, this converges to an equilibrium normal distribution of mean θ/α and variance $\sigma^2/2\alpha$. This does not mean that the process r_t converges but the distribution of r_t converge to the normal distribution.

3 Premium Calculation

We shall define the present value function, Z_t, by

$$z_t = b_t \, v_t \tag{7}$$

Here b_t is benefit function and v_t is a interest discount factor from the time of payment claim to the time of policy issue. The v_t determine from r_t, investment return as

$$v_t = \prod_{s=1}^{t} \frac{1}{\left(1 + r_s\right)}, \qquad \forall s, r_t > -1 \tag{8}$$

t is length of the interval from issue to death. Therefore, z_t is the present value, at policy issue, of the benefit payment. The elapsed time from policy issue to death of the insured is the insured's future-lifetime random variable, denote as T and V is random variable of interest discount factor. Thus, the present value, at policy issue, of benefit payment is the random variable z_T. We shall denote this random variable by Z and base the model for the insurance on equation.

$$Z = b_t \, V_T \tag{9}$$

An n-year term life insurance provide for a payment only if the insured dies within the n-year term of an insurance commencing at issue. If a unit is payable at the moment of death of (x), then

$$b_t = \begin{cases} 1 & t \le n \\ 0 & t > n \end{cases} \tag{10}$$

Then expectations of the present-value random variable Z is called the net single premium

$$\overline{A}^{1}{}_{x:\overline{n}|} = E[Z] = \int_0^{\infty}\int_0^{t} b_t\, v_s\; dF_r(t)\, dF_X(t) \tag{11}$$

if we make a simple approximation to equation (4) by assuming that every claim will pay at the end of year, then we will have the net single premium or present value of probability claim amount.

$$A^{1}{}_{x:\overline{n}|} = E[Z] = \sum_{t=0}^{\infty}\sum_{s=0}^{t} b_t\, v_s\; {}_{t|}q_x \tag{12}$$

We use net premium principles and standard deviation principles (Kaas; 2001) to calculate the net premium

1. Net Premium Principles : NP = E[Z]
2. Standard Deviation Premium Principles : NP = E[Z] + φ σ[Z]

where φ is positive number.

4 Monte-Carlo Simulation

As we know the distribution of r_t converge to the normal distribution but equation (8) only accept r_t more than negative one (in Indonesian situation more than zero). It's not easy to handle this situation. The standard approach to handle this situation in practice is to use simulation. Let we consider the equation (4) with discrete version

$$\Delta r = (\theta - \alpha\, r_t)\Delta t + \sigma\,\varepsilon\sqrt{\Delta t} \tag{13}$$

where ε is normal distribution with mean zero and variance 1. In the limit, as Δt goes to zero. We note that

$$(\Delta r)^2 = \sigma^2\,\varepsilon^2\Delta t + \text{terms of higher other in } \Delta t$$

Now we have

$$r_{t+1} = r_t + (\theta - \alpha\, r_t)\Delta t + \sigma\,\varepsilon\sqrt{\Delta t} \tag{14}$$

if we make a simple approximation to equation (4) by assuming that every claim will pay at the end of year and $\Delta t = 1$, then we will have

$$r_{t+1} = r_t + (\theta - \alpha\, r_t) + \sigma\,\varepsilon \tag{15}$$

We consider discount factors as equation (8).

The Monte-Carlo simulation procedure for n years insurance coverage and m replication is as follows :

1. Generate t random numbers from normal standard distribution.
2. Next find sequence r_1 , r_2 , … , r_n using equation (15) and discount factors v_1 , v_2 , … , v_n using equation (8).
3. Calculate the net single premium by equation (12).
4. By m replication, we can have the m possible value of net single premium

Suppose the *m* replication is stochastic independent then we can have an average value as expectations of the present value random variable Z (Ross; 1996).

Let us denote the *m* possible value of net single premium are $X_1, X_2, \ldots, X_m$, because for each net premium is one of the possible value of the net premium then $X_1, X_2, \ldots, X_m$ are stochastic independent random variable with mean Φ and variance Ω^2. Although the sample mean X and variance S^2

$$\overline{X} = \sum_{i=1}^{m} X_i / m \qquad S^2 = \sum_{i=1}^{m} \left(X_i - \overline{X}\right)/(m-1)$$

are effective estimator of mean Φ and variance Ω^2., thus from the central limit theorem, it follows that for large m

$$\sqrt{n}\,\frac{\left(\overline{X}-\Phi\right)}{S} \sim N(0,1) \tag{16}$$

By using the bootstrapping technique for estimating mean square error from the sample value of net single premium $X_1, X_2, \ldots, X_m$. We can estimate the underlying probability distribution function F of net single premium by the so-called empirical probability distribution function F_e , the probability that present value of claim or net single premium value is less than or equal to x that is

$$F_e(x) = \frac{Numb\ of\ i,\ X_i \le x}{m} \tag{17}$$

If m is large then F_e is "close" to F. In probability theory, we know the strong law of large numbers implies that F_e converges to F as $m \to \infty$ with probability 1.

5 Case Study : Five-Year Life Insurance

In this paper we will examine net single premium of five-year life insurance with benefit 1000 at the moment of death of (x) and probability risk 0.001 for each years coverage. This includes the profit sharing of investment 70% to policy owner at the end of policy years. We assume the parameter models are $\theta = 0.06$, $\alpha = 0.5$, $\sigma = 0.15$ and initial/current investment return is 9% per annual.

The most important technique in Monte-Carlo simulation is to get "large number" sample. Now, we use 500 replication to get "large number", its enough number to get the approximation of expectation of random variable Z.

First step of Monte-Carlo Simulation is generate $n\,m$ random numbers (n years and m replication), in this case we generate 2500 random number for 5 years with 500 replication. By equation (15), we generate 2500 number of ε from normal distribution with mean 0 and variance 1.

Table 1. Random Numbers ~ N(0,1)

	Replication				
Year	**1**	**2**	**3**	**4**	**5**
1	0.64	-0.92	0.25	-1.47	-0.90
2	-1.17	0.74	1.76	1.08	-0.18
3	0.08	-1.07	-0.48	-0.45	-0.54
4	-0.62	-0.67	0.16	0.34	0.49
5	1.90	-2.14	-0.22	1.75	-1.66

Using equation (15), We can find sequence { r_t } of investment return for 5 years and 500 replications. Table 2 is investment return from table 1.

Table 2. Investment Returns					
	Replication				
Year	**1**	**2**	**3**	**4**	**5**
0	9.0%	9.0%	9.0%	9.0%	9.0%
1	11.5%	9.1%	10.9%	8.3%	9.2%
2	10.0%	11.7%	14.1%	11.8%	10.3%
3	11.1%	10.2%	12.3%	11.2%	10.3%
4	10.6%	10.1%	12.4%	12.1%	11.9%
5	14.2%	7.9%	11.9%	14.7%	9.5%

Using equation (8) and (12), We can find present value of insurance benefit for each year. This value after we take the profit sharing factor for the insurance company, in this case the investment return will be given to the company 30% and the 70% to the policy owner.

Table 3. Present Value of Insurance Benefit					
	Replication				
Year	**1**	**2**	**3**	**4**	**5**
1	0.93	0.94	0.93	0.95	0.94
2	0.87	0.87	0.85	0.87	0.88
3	0.80	0.81	0.78	0.81	0.82
4	0.75	0.76	0.72	0.75	0.75
5	0.68	0.72	0.66	0.68	0.71

Sum of present value of insurance benefit for each year is the net premium of five years insurance coverage. Let's see table 4, 5 possible net premium of five-year life insurance.

Table 4. Five Net Premium of 5 Years Life Insurance					
	1	**2**	**3**	**4**	**5**
NP	4.02	4.10	3.93	4.05	4.10

Let us consider table 5, expectation of investment return r_t is close to the mean of our model θ / α =12%.

Table 5, Statistic of Invesment Return	
Mean of sample =	**11.34%**
Variance sample =	**0.0003**
Standard Deviation =	**0.0175**

Consider table 6, expectation of net premium with 500 replication

Table 6. Statistic of Net Premium from Random Variable Z	
Mean of sample =	**4.0234**
Variance sample =	**0.0086**
Standard Deviation =	**0.0926**

Let us consider figure 1, a histogram of 500 possible net premium of five-year life insurance with death benefit 1000. Using bootstrapping technique we can approximate the probability that net premium will be cover the possible of the investment return happen in five years. If we take x (positive value) as a premium then we have the number of net premium less or equal than x, denote y, It mean x possible to cover $y/500$ of the possible investment return happen in five years.

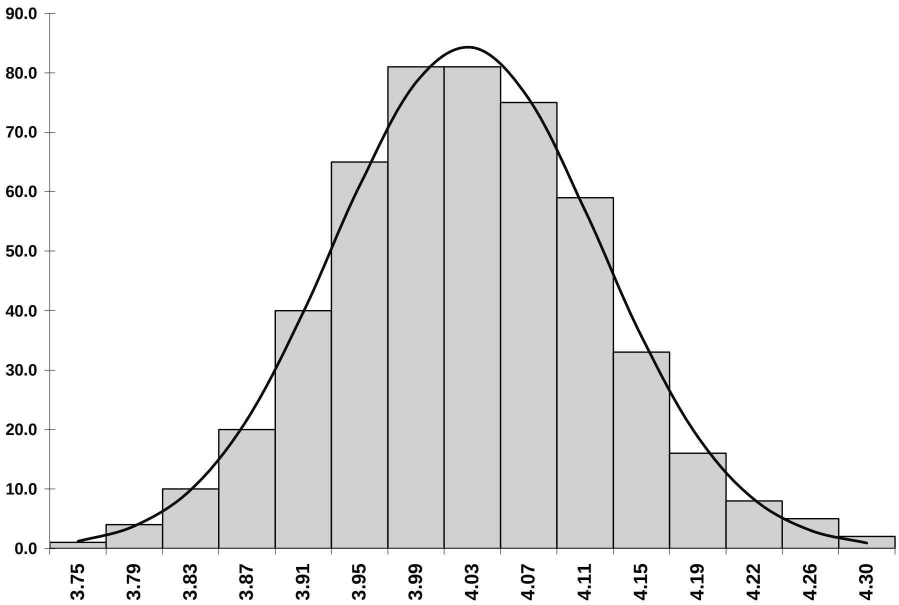

Figure 1. Histogram of 500 possible net premium.

Consider table 7, Then net premium of five-year life insurance use stochastic models and the net premium principles and standard deviation principles (Kaas, 2001), where φ is factor that we choose as safety margin of the net premium. The net premium use various safety margin and relates to the probability that net premium can cover the possible investment return happen in five years.

Table 7. Net Premium Five Years Life Insurance use stochastic models		
Factor φ	***Premium x - value***	**Prob.**
0.0	4.02	51.6%
0.5	4.07	69.4%
1.0	4.12	86.0%
1.5	4.16	93.4%
2.0	4.21	97.0%
2.5	4.25	99.2%
3.0	4.30	99.6%

Now we compare this technique to the classic technique which conventional product has been developed. Let us consider table 9, net premium rate which various fixed interest rate from 6% to 14%.

Table 8. Net Premium Five Years Life Insurance use classic method		
Int. Rate	**Premium Rate**	**Prob.**
6%	4.43	100.0%
7%	4.34	100.0%
8%	4.26	99.2%
9%	4.18	95.4%
10%	4.10	80.6%
11%	4.02	51.6%
12%	3.95	21.6%
13%	3.88	4.8%
14%	3.81	1.0%

When we assume, the initial or current investment return is 9% per annual. Interest rate from 6% to 9% have net premium more than 4.02 (net premium using stochastic models). Interval from 12% to 14% have net premium rate less than 4.02 but starting the first year company has negative spread (from -3% to -5% pa) and in this interval the premium can cover maximum 21% of the possible investment return happen in five years. It is clear that using stochastic models, the five years life insurance syariah will be cheaper and more competitive than conventional product. Moreover insurance company who sells syariah product does not have the negative spread and more solvency than sell conventional product.

Finally stochastic models can be applied into syariah principles and using this method the premium can be more competitive than conventional method. We hope the Indonesian regulator will accept this method as one of standard premium calculations method of the syariah product. It is more interesting concept when we use stochastic model to define the reserve valuation method for syariah product and most important study is how to measure the solvency of syariah insurance company. ■

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OCKE KURNIANDI: Associate Actuary, PT. MAA Life Assurance
Kawasan Bisnis Granadha, Jl. Jenderal Sudirman Kav 50 Lt 9,
Jakarta 12930, Indonesia.
Phone: +6221 255 39466 / Fax: +6221 255 39467
E-mail: ocke@maa.co.id