MODELING NATURAL CATASTROPHIC RISK AND ITS APPLICATION

Y. Uchiyama, S. Watanabe

Nissay Dowa General Insurance, Japan

Abstract. In the situation that huge natural disasters such as typhoons and earthquakes attack Japan frequently, non-life insurance companies have played the great role in insurance service and its further expansion will be requested in the future. However, a huge natural disaster might make fails non-life insurance companies or drive them into extremely unstable condition momentarily when their risk management is insufficient or their financial strength is weak. This report presents an overview of natural catastrophic risk models. Their applications in Japan also described.

Key-words: Engineering risk model, Statistical risk model, Secondary uncertainty

1 Introduction

Insurance products that cover natural catastrophic risks are becoming more popular in Japan, while the natural catastrophic risk increases globally in recent years.

Quantitative analysis of the impact of the natural catastrophic risk and its application for proper insurance pricing, management of accumulated risks, review of capital adequacy and optimization of hedges etc. is a critical issue for non-life insurance companies because the natural catastrophic risk has the possibility to exert an significant influence on them.

The purpose of this paper is to introduce the application of the natural catastrophic risk model in Japan.

The contents of this paper are as follows. In Chapter 2 and Chapter 3, the recent trend of the natural catastrophic risk and the feature of the natural catastrophic risk in Japan are discussed. In Chapter 4, we introduce a brief overview of the natural catastrophic model. The numerical example with Japanese empirical data is also shown. In Chapter 5, we introduce the application of the natural catastrophic risk model in Japan. Finally, the model risk and the parameter risk in the natural catastrophic risk model are discussed.

2 Increase of natural catastrophic risk

There is also a primary factor of the unusual weather due to terrestrial warming, large-scale natural disaster increases on a worldwide scale.

Among the world's largest 20 cases of insurance payment for natural disasters in 1970-2004, 17 cases occurred after 1990. (Depending on the Swiss re-insurance company "sigma" magazine)

The natural disaster such as Typhoon Mireille (No.19 1991), Typhoon Bart (No.18 1999), Typhoon Songda (No.18 2004) and Great Hanshin Earthquake in Kobe (M 7.2 1995) that occurred in Japan are also included in these 17 cases.

Moreover, among the largest 37 cases in 1970-2004, 34 cases occurred after 1987.

Natural catastrophes: The most costly insurance losses 1970-2004

Insured loss*	Date (start)	Event	Country
21,542	1992/8/23	Hurricane Andrew	US, Bahamas
17,843	1994/1/17	Northridge earthquake(M6.6)	US
11,000	2004/9/2	Hurricane Ivan; damage to oil rigs	US, Caribbean: Barbados
8,000	2004/8/11	Hurricane Charley	US, Caribbean: Cuba
7,831	1991/9/27	Typhoon Mireille/No.19	Japan
6,639	1990/1/25	Winterstorm Daria	France, UK et al
6,578	1999/12/25	Winterstorm Lothar over Western Europe	France, CH et al
6,393	1989/9/15	Hurricane Hugo	Puerto Rico, US et al
5,000	2004/8/26	Hurricane Frances	US, Bahamas
5,000	2004/12/26	Seaquake(Mw9.0), tsunamis in Indian Ocean	Indonesia, Thailand et al
4,988	1987/10/15	Storm and floods in Europe	France, UK et al
4,613	1990/2/25	Winterstorm Vivian	Western/Central Europe
4,582	1999/9/22	Typhoon Bart/No.18	Japan
4,091	1998/9/20	Hurricane Georges	US, Caribbean
4,000	2004/9/13	Hurricane Jeanne; floods, landslides	US, Caribbean: Haiti et al
3,585	2004/9/6	Typhoon Songda/No.18	Japan, South Korea
3,361	2001/6/5	Tropical storm Allison; rains, flooding	US
3,292	2003/5/2	Thunderstorms, tornadoes, hail	US
3,065	1995/1/17	Great Hanshin earthquake (M7.2) in Kobe	Japan
2,722	1999/12/27	Winterstorm Martin	Spain, France, CH
2,677	1999/9/10	Hurricane Floyd; floods	US, Bahamas et al
2,603	1995/10/1	Hurricane Opal	US, Mexico
2,535	2002/8/6	Severe floods across Europe	Europe
2,358	1991/10/20	Forest fires that spread to urban areas, dought	US
2,347	2001/4/6	Hail, floods and tornadoes	US
2,289	1993/3/10	Blizzard, tornadoes	US, Mexico, Canada
2,154	1992/9/11	Hurricane lniki	US, North Pacific Ocean
1,958	1979/8/29	Hurricane Frederic	US
1,927	1996/9/5	Hurricane Fran	US
1,916	1974/9/18	Tropical cyclone Fifi	Honduras
1,883	1997/7/4	Floods after heavy rain in Central Europe	Poland, Czech Republic, D et al
1,860	1995/9/3	Hurricane Luis	Caribbean
1,759	2002/4/27	Spring storm with several tornadoes	US
1,746	1988/9/10	Hurricane Gilbert	Jamaica, Mexico et al
1,730	2003/9/18	Hurricane Isabel	US, Canada
1,701	1999/12/3	Winterstorm Anatol in Western and Northern Europe	Denmark, Sweden et al
1,684	1999/5/3	Series of more than 70 tornadoes in the Midwest	US

* Property and business interruption, excluding liability and life insurance losses
(in US$ Million, indexed to 2004)

(Source) Swiss Re, sigma No.1/2005

Especially in recent years, large-scale damages have been caused by earthquake or by wind and flood in the Asian region such as India, Afghanistan and China.

In the Asian area in spite, in recent years every year, the damage where the number of deceased or missing exceeds several thousand has been recurring in spite of great efforts of disaster prevention and reduction by the government of each country.

3 Natural catastrophic risk in Japan

3-1 Feature of the natural catastrophic risk in Japan

Because of the geographic factors and the social environmental factors, Japan is a country that relatively is mainly exposed to natural catastrophic risk.

(1) Geographical factors

Japan, being the island country that is located to the circum-Pacific seismic zone, is prominent earthquake country even in the world. Because the country has been categorized to the boundary of the oceanic plate and the continent plate, enormous earthquake of the plate boundary type that occurs due to the downgrade being packed of the plate occurs. In addition, also crust interior of a country shaking of the inland limits that originate in the motion of the plate occurs. In addition, because being surrounded by the sea, the shoreline to be long has become complicated depending upon the inlet. Therefore, in the case of earthquake, the big damage by the tidal wave is easy to occur.

In addition, our country is located to the circum-Pacific volcanic zone and there are 108 active volcanoes, which are approximately 10% of the entire world. The volcanic disaster due to the eruption phenomenon and the volcanic characteristic earthquake with Mt. Usu and Miyake island occurred even in 2000, brought big damage.

Furthermore, because most of the country has been categorized to the temperate region, four seasons appear clearly. As various weather phenomena of four seasons, Rain front is stagnant near Japan from the end of spring through the summer and the typhoon goes north from the tropical limits from the summer extending through the fall. Each of them has exerted big influence on Japan weather.

The rain front the abundant rainfall is brought, the typhoon landing or approaching in Japan, or, brings the rainstorm. Furthermore, in the area of the Sea of Japan side there is the strong chill that comes from the Siberian continent in the winter. It is also the mass snowfall area even in the world.

As for our country, because of the steep topography, there are many mountains, valleys and cliffs and rivers are scarp considerably. Therefore, once it rains heavily, river flow increases suddenly the disaster by the flood is likely to happen. Furthermore, also earth and sand disaster, the flood of rocks and mud or landslide, is also likely to occur. These are also pulled up with earthquake.

(2) Social environmental factors

Although Japan is a small country (380,000km^2), It has the population of one hundred and 20 million. It is overcrowded in plains where 90 percent of the population live in less than 40 percent of the area of the country.

Many of the plain section expand in the vicinity of the seashore and the rivers. Especially, because the population is centering in the alluvial plain that is lower than rivers water level, at the time of typhoon or heavy rain the inundation of the rivers causes extensive damage easily.

Furthermore, there is a feature of national condition such that high-level land utilization is done, when for population overcrowding disaster occurs, the damage becomes big ones.
Especially, because of the development of urbanization, the victim by landslide reached the point where big ratio is occupied in the victim due to natural disaster

3-2 Tendency of recent natural disasters in Japan

As for the natural disasters that swept our country in the past, all of the superior 10 rank of the insurance payment due to wind and flood disaster are those which occurred after 1990. Rank 2, rank 5 and rank 6 are the typhoons that occurred in 2004. Same as the case seen in the worldwide scale, enormous natural disaster concentrates in these 15 years and it exists in the increasing tendency.

Claims Paid for Natural Disasters (Typhoon or Windstorm) In Japan

Rank	**Claims Paid** (incl. Estimates) (US$ Million)	**Date (start)**	**Name of Disaster**	**Place**
1	5,679	1991/9/26	Typhoon No.19 (Mireille)	Nationwide
2	3,823	2004/9/4	Typhoon No.18 (Songda)	Nationwide
3	3,147	1999/9/21	Typhoon No.18 (Bart)	Kumamoto, Yamaguchi, Fukuoka, etc.
4	1,600	1998/9/22	Typhoon No.7	Kinki
5	1,292	2004/10/20	Typhoon No.23 (Tokage)	Western Part of the Nation
6	1,175	2004/8/30	Typhoon No.16 (Chaba)	Nationwide
7	1,030	2000/9/10	Downpour, Sep. 2000	Aichi, etc.
8	977	1993/9/3	Typhoon No.13	Kyushu, Shikoku, and Chugoku
9	700	2000/5/24	Hailstorm	Chiba and Ibaraki
10	365	1990/9/17	Typhoon No.19	Nationwide

(Note) Figures in 2004 include estimated claims and are subject to change
Conversion rate:￥1,000=US$10
(Source) The General Insurance Association of Japan (www.sonpo.or.jp)

Because Typhoon Mireille (No.19/1991) was the biggest typhoon in damage scale these 20 years, the establishment of system that can withstand the same scale typhoon as Typhoon Mireille, is the common goal of government side and insurance company's side.

However, as liberalization progressed further, risk management and the self-responsibility of each insurance company was required, the necessity of the system that can withstand a larger typhoon came to be emphasized.

In 2005 the required level of provision against typhoon disaster was revised from Typhoon Mireille (No.19/1991), which reappears in 25 to 30 years, to Typhoon Isewan (1959), which reappears in 70 years.

In addition, while the scale of typhoon has been focused, in 2004 the number of typhoon that swept our country was 10 and was much more than the average, 2.6.

In 2004 Typhoon Chaba (No.16), Typhoon Songda (No.18) and Typhoon Tokage (No.23) screwed up violence especially, and the total amount of insurance payment due to the typhoon disaster was more than that of 1991.

This way, two types of danger exists in typhoon disaster, one is the danger that the enormous typhoon would attack the country and the other is the danger that frequency of landing of the typhoon would increase due to the fact that fluctuation of a some terrestrial environment changes typhoon course delicately,

3-3 Change of natural catastrophic risk cover in Japan

History of the natural catastrophic risk cover in fire insurance was relatively new, in the dwelling-house comprehensive insurance 1961 and the store comprehensive insurance 1962, when natural disaster risk, storm and flood, was newly assured. However, amount of the payment was limited, at the beginning, only for total loss, and fixed rate (3%) payment of the insurance amount was paid.

After that, it was gradually improved, payment ratio for natural disaster revised from 10% of the insured value to 30%, furthermore from 30% co-insurance to 50% co-insurance, in 1984 as for flood disaster it is modified to 70% co-insurance.

As for the cover for windstorm, hailstorm or snow disaster, co-insurance condition abolished. The limit of payment was modified to 2000, 6000, 12000, 48000, 100000 US$ from 1500 US$ at the beginning, at last the limit was abolished in 1984. (Conversion rate: ¥1,000=US$10)

Also the ratio of the comprehensive insurance that covers wind and flood disaster rose and reached to 40% in 1975. Afterwards in 1984 it had reached the point where it exceeds 50%.

After that, each insurance company modified original fire line that covers typhoon and floods disaster without payment reduction, for example co-insurance condition, and sells it positively, which increases the amount of payment for typhoon and flood disaster.

4 Natural catastrophic risk model

As mentioned in introduction, quantification of natural catastrophic risk is essential for the non-life insurance company. Natural catastrophic risk models estimate loss amount of an insurance company caused by natural catastrophic hazards as a probability distribution. The natural catastrophic risk model can be classified into the "engineering risk model" and the "statistical risk model".

4-1 Engineering risk model

The engineering risk model is the model that calculates the distribution of loss amount caused by natural catastrophic hazards applying scientific theory such as seismology, meteorology, etc. and the terms and conditions of insurance contracts. [2] shows typical modules of engineering risk models.

Event generation: where events with certain features (such as quake intensities and locations, windstorm windfields and paths over land) are simulated

Mitigation: where allowance is made for any mitigation features such as flood defences

Damage: where the simulated intensity of the event is converted into an amount of damage to the insured property.

Insurance: where the terms of the insurance contract are applied in order to calculate the loss to the insurance contract.

Since engineering risk models are introduced in many papers, we do not mention them any further.

4-2 Statistical risk model

The statistical risk model is the model that builds up loss distribution using statistical technique without considering mechanisms of natural hazards. It can be said that the statistical risk model is based on top-down approach as contrasted with the engineering risk model that is on bottom-up approach.

The statistical risk model can be divided into two types. One is the model that deals with the frequency and the severity of natural catastrophic hazards separately. The other is the model that deals with the sum total of loss amount as a whole. The latter can be considered as a special case of the former whose frequency is constant 1. We discuss the frequency distribution and the severity distribution respectively.

(1) Frequency distribution

As a frequency distribution, Poisson distribution is commonly used. Poisson distribution is not appropriate when the mean is not equal to variance. For example, in case of variance > mean, negative binomial distribution should be considered as a candidate. Whereas the natural catastrophic hazards which is dealt with in a model rarely occurs. Thus there are rarely enough data to overturn the hypothesis of mean = variance.

It is necessary to note assumptions of independence and constancy. As for windstorm and flood, there were three events where the paid loss exceeds a hundred billion Yen in 2004. In contrast, there were only four in the last 20 years. If we assume independence and constancy, it means that quite rare events occurred. Concerning independence, we cannot rule out the possibility that ENSO (El Niño Southern Oscillation) have an impact on them. In Japan, it is not recognized that the number of occurrences and route of the typhoon change in the year of El Niño.[1] However, when the fact that influence of the El Niño in medium-high latitude is indirect is considered, it should be examined the possibility that the influence appears with some time lags. In addition, concerning constancy, it is pointed out the possibility such as increase of the precipitation, increase of rainstorm strength and rise sea level caused by global warming, which will impact on wind and flood damages. It means that the frequency of the catastrophic hazards becomes higher. It is necessary to carefully observe future tendency.

---

[1] Japan Meteorological Agency
(http://www.data.kishou.go.jp/climate/elnino/faq/faq8.html#4)

(2) Severity distribution

a. Pareto distribution

As the severity distribution for non-Cat loss, the gamma distribution and the log normal distribution are often used. On the other hand, for Cat loss, it is general to use the Pareto distribution

$$\Pr(X \le x) = 1 - \left(\frac{\beta}{x}\right)^{\alpha}, \ (x \ge \beta, \alpha, \beta > 0)$$

so that occurrence probability do not approach to 0 quickly.[2]

For example, the Gutenberg-Richter Law

$$N(M \ge m) = 10^{a-bm}$$

is known as displays the relationship between magnitude M and frequency of occurrence N . Suppose that the lower limit of magnitude is m_l , then

$$\Pr(M \le m) = \frac{N(M \ge m_l) - N(M \ge m)}{N(M \ge m_l) - N(M \ge \infty)} = \frac{10^{a-bm_l} - 10^{a-bm}}{10^{a-bm_l}} = 1 - \left(\frac{10^{m_l}}{10^{m}}\right)^{b},$$

that is,

$$\Pr(10^{M} \le 10^{m}) = 1 - \left(\frac{10^{m_l}}{10^{m}}\right)^{b}.$$

This means that 10^{M} follows a Pareto distribution.

It is possible to judge visually to some extent whether fitting the Pareto distribution for observed data is suitable or not.

Suppose

$$y = \Pr(X > x) = \left(\frac{\beta}{x}\right)^{\alpha}.$$

Then the relationship between $\log x$ and $\log y$ can be expressed as

$$\log y = -\alpha \log x + \alpha \log \beta .$$

---

[2] The right tail of distribution is thick in order of gamma distribution < log normal distribution < Pareto distribution. It is found by examining the limit of ratio of probability density function.(See [1])

Therefore, it is possible to judge whether severity distribution follows Pareto distribution or not by inspecting whether the plotted $\log x$ and $\log y$ (log - log plot) is on a straight line.

Furthermore, assume that the regression line over the actual data is

$$\log y = a \log x + b\,.$$

Then we can obtain the parameters as follows by comparing the coefficients respectively:

$$\alpha = -a\,, \beta = e^{-b/a}\,.$$

In addition, we can substitute the rank z ($y = z/n$) in the n events in observation period for $y = \Pr(X > x)$. In this case, the regression line is

$$\log z = -\alpha \log x + \alpha \log \beta + \log n\,,$$

thus, the parameters are shown as follows:

$$\alpha = -a\,, \beta = (e^{b}/n)^{-1/a}\,.$$

b. Generalized Pareto distribution

The generalized Pareto distribution is often used as a severity distribution, too. Suppose that u is a threshold. In most distributions, the conditional probability

$$\Pr(X \le x + u \mid X > u) = \frac{\Pr(X \le x + u) - \Pr(X \le u)}{1 - \Pr(X \le u)}$$

approaches to

$$1 - \left(1 + \xi \frac{x}{\sigma}\right)^{-1/\xi}, x \ge 0$$

namely the distribution function of generalized Pareto distribution as u increases.[3] Therefore, generalized Pareto distribution is often used as a severity distribution.

Using this, the distribution function of a severity distribution is described as follows:

---

[3] Pickands and Balkema-De Haan's theorem.

$$\Pr(X \le x) = \begin{cases} F_1(x) & (x < u) \\ F_1(u) + (1 - F_1(u)) \times \left\{ 1 - \left(1 + \xi \dfrac{x-u}{\sigma} \right)^{-1/\xi} \right\} & (x \ge u) \end{cases},$$

where $F_1(x)$ is the distribution function under threshold.

Same as case of Pareto distribution, it is possible to judge visually whether fitting the generalized Pareto distribution for observed data is suitable or not.

Consider mean excess function $e(u)$ (i.e. conditional expectation $E(X-u \mid X > u)$).

$$e(u) = E(X-u \mid X > u) = \frac{\sigma + \xi u}{1-\xi}, 0 < \xi < 1$$

That is a linear function of u , thus it is possible to judge whether severity distribution follows a generalized Pareto distribution or not by inspecting whether the plotted u and $e(u)$ (mean excess plot) is on a straight line.

Furthermore, assume that the regression line over the actual data is

$$e(u) = au + b,$$

then we can obtain the parameters as follows by comparing the coefficients respectively.

$$\xi = \frac{a}{1+a}, \sigma = \frac{b}{1+a}.$$

c. Note

In considering a severity distribution, we should note independence and stability similarly as discussed in the section of frequency distribution. In addition, it should be taken into account that influences with inflation, contract quantity or changes in cover or construction code.

(3) Correlation of Risks

In natural catastrophic hazards, enormous damage may occur simultaneously across multiple lines. When many buildings are damaged by a typhoon, it is likely that there are a large number of automobiles submerged.

Suppose that X_1, X_2 are random variables, u_1, u_2 are thresholds respectively and $F(z_1, z_2)$ is the joint distribution function of (Z_1, Z_2) = ($\max(X_1, u_1), \max(X_2, u_2)$).

Using copula, $F(z_1, z_2)$ can be expressed as follows:

$$F(z_1, z_2) = C(F_1(z_1), F_2(z_2)),$$

where F_1, F_2 are the marginal distribution function of Z_1, Z_2.

[3] states that $C(t_1, t_2)$ converges in following copula called extreme value copula as $u_1, u_2 \to \infty$.

$$C(t_1, t_2) = \exp\left\{-V\left(-\frac{1}{\log t_1}, -\frac{1}{\log t_2}\right)\right\},$$

where $V(z_1, z_2) = \int_0^1 \max\{s{z_1}^{-1}, (1-s){z_2}^{-1}\} dH(s)$.

H is a measure on [0,1] satisfying $\int_0^1 s dH(s) = \int_0^1 (1-s) dH(s) = 1$.

Since extreme value copulas exist innumerably depending on H, it is necessary to select appropriate ones from among those. In practice, Gumbel copula is used commonly. α is a parameter which represents the degree of correlation. By using α, Kendall's τ is expressed as

$$\tau = 1 - \alpha^{-1}.$$

In contrast, we can estimate the parameter α by Kendall's τ obtained from observed data.

$$\alpha = \frac{1}{1-\tau}$$

4-3 Example of statistical risk model

In this section, we will show a numerical example of the calculation of aggregate loss distribution of the Japanese windstorm and flood risk by the statistical risk model. First, we estimate the loss distributions for fire and the other lines separately, and then we calculate the aggregate loss distribution using Gumbel copula.

(1) Data

We analyzed the largest 14 windstorms and floods in the paid loss between 1985 and 2004 in Japan. (Paid loss is the total amount of the companies affiliated with the General Insurance Association of Japan)

Rank	Name	Date	Claims Paid (US$ m)		
			Fire,Misc.	Other Lines	Total
1	Typhoon No.19	1991.9.26～28	5,225	454	5,679
2	Typhoon No.18	2004.9.4～8	3,564	259	3,823
3	Typhoon No.18	1999.9.21～25	2,847	300	3,147
4	Typhoon No.7	1998.9.22	1,514	86	1,600
5	Typhoon No.23	2004.10.20	1,113	179	1,292
6	Typhoon No.16	2004.8.30～31	1,037	138	1,175
7	Downpour, Sep. 2000	2000.9.10～12	447	583	1,030
8	Typhoon No.13	1993.9.3	933	44	977
9	Hailstorm	2000.5.24	372	328	700
10	Typhoon No.19	1990.9.17～20	324	41	365
11	Typhoon No.17	1991.9.14～15	339	8	347
12	Typhoon Nos.12,13,14	1985.8.29～9.2	281	30	311
13	Typhoon No.7	1993.8.10	232	65	297
14	Typhoon No.26	1994.9.29～30	245	25	270

(Note) Conversion rate: ¥1,000=US$10
(Source) The General Insurance Association of Japan
(http://www.sonpo.or.jp/e/index.html)

(2) Frequency distribution

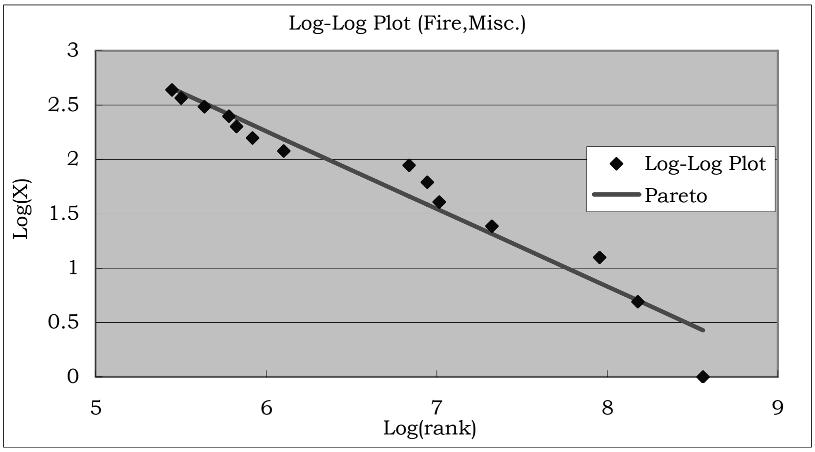
Since there are 14 hazards in 20 years, we assumed that the frequency follows the Poisson distribution with mean 0.7(=14/20).

(3) Severity distribution

a. Fire

The following chart is the result of plotting the logarithm of rank ($\log z$) and the logarithm of paid loss ($\log x$).

(Log-Log plot)



The regression line is

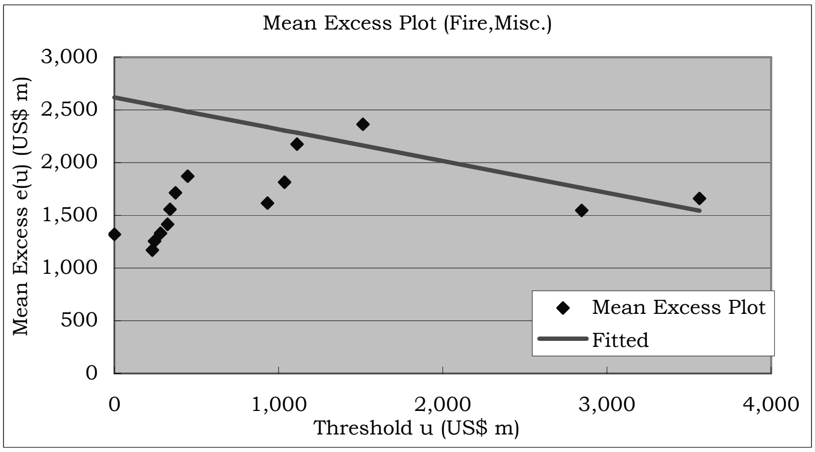
$$\log z = -0.714413 \log x + 6.546557,$$

then the parameters of the Pareto distribution are estimated as follows:

$$\alpha = 0.714413, \beta = 237.347.$$

Next, we consider the mean excess plot (the following chart), which plots the threshold and the corresponding mean excess.

(Mean excess plot)



From the chart, it can be shown the mean excess plot is on a straight line in $u \geq 1514$. The regression line is

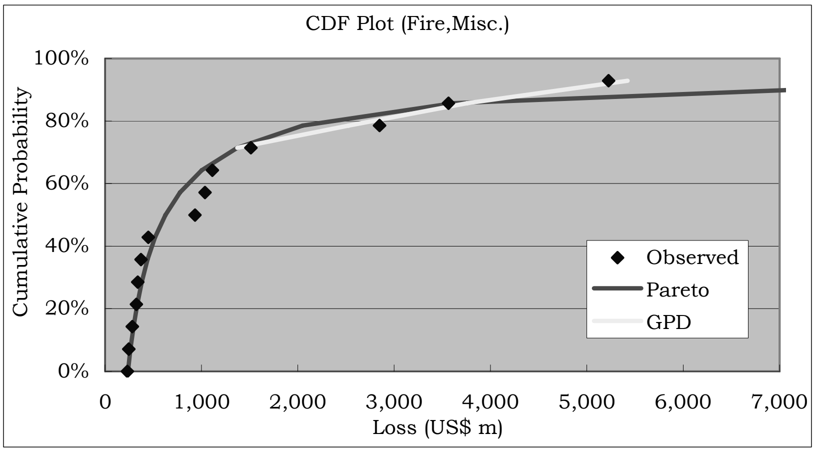
$$e(u) = -0.301241u + 2617.570,$$

thus the parameters of the generalized Pareto distribution are estimated as follows.

$$\xi = -0.431108, \sigma = 3746.025.$$

The following chart is the cumulative probability plot of observed data and the result of fitting the Pareto distribution and the generalized Pareto distribution.

(CDF plot)



The generalized Pareto distribution fits better than the Pareto distribution in the right tail of the distribution. Therefore, we adopt the distribution that connects the both distributions at $u = 1514$.

$$F_1(x) = \begin{cases} G_1(x) & (x < u_1) \\ G_1(u_1) + (1 - G_1(u_1)) \times H_1(x - u_1) & (x \geq u_1) \end{cases}$$

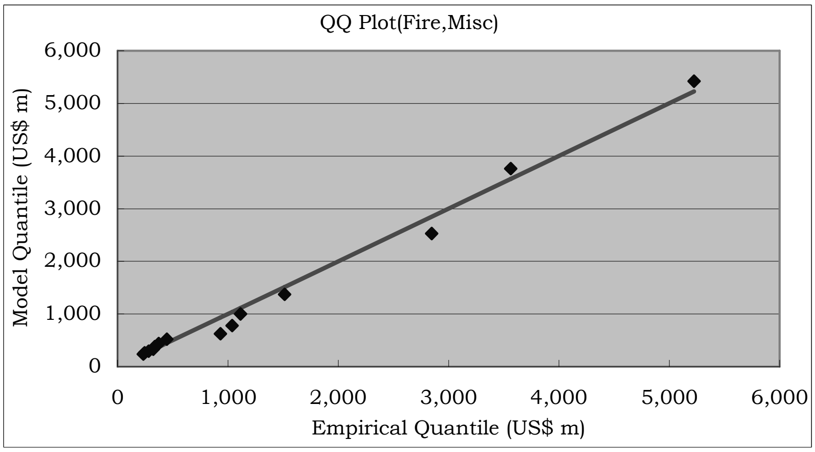
$$G_1(x) = 1 - \left(\frac{\beta_1}{x}\right)^{\alpha_1}, H_1(x) = 1 - \left(1 + \xi_1 \frac{x}{\sigma_1}\right)^{-1/\xi_1}$$

$$u_1 = 1514\,,\ \alpha_1 = 0.714413\,,\ \beta_1 = 237.347\,,$$

$$\xi_1 = -0.431108\,, \sigma_1 = 3746.025$$

The following chart is the Q-Q plot of the severity distribution adopted.

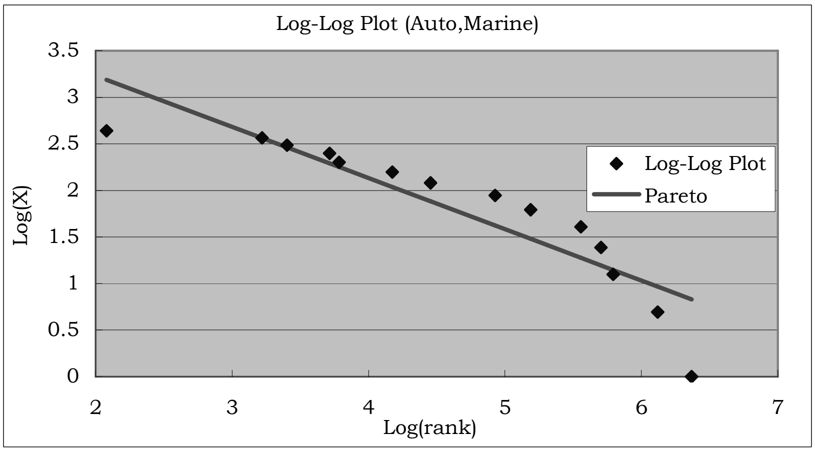
(Q-Q plot)



b. Other lines

Same as the case of fire, we adopt the distribution that connects a Pareto distribution and a generalized Pareto distribution as the severity distribution for the other lines.

(Log-Log plot)



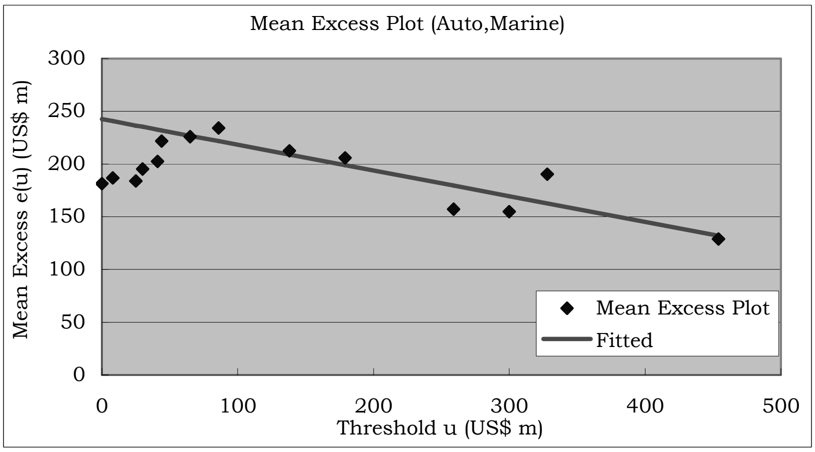
The regression line is

$$\log z = -0.550134 \log x + 4.333153,$$

then the parameters of the Pareto distribution are estimated as follows:

$$\alpha = 0.550134, \beta = 21.746.$$

(Mean excess plot)



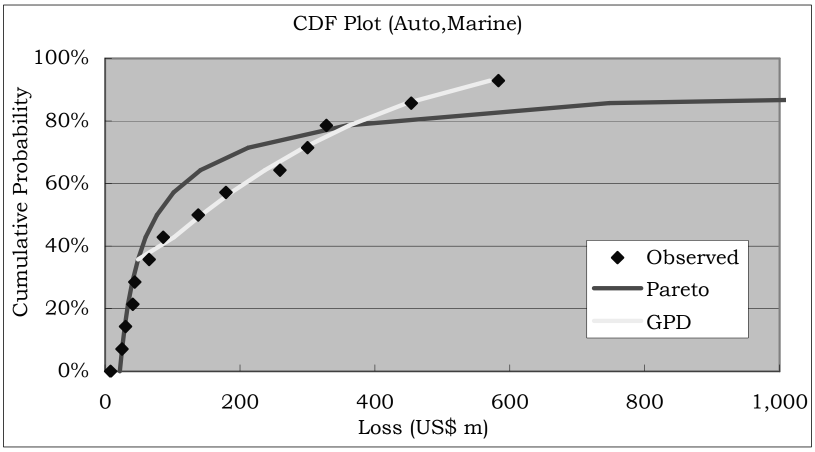
From the chart, it can be shown the mean excess plot is on a straight line in $u \geq 65$. The regression line is

$$e(u) = -0.243975u + 242.659$$

thus the parameters of the generalized Pareto distribution are estimated as follows:

$$\xi = -0.322708, \sigma = 320.967.$$

(CDF plot)



Same as the case of fire, we adopt the distribution that connects the both distributions at $u = 65$.

$$F_2(x) = \begin{cases} G_2(x) & (x < u_2) \\ G_2(u_2) + (1 - G_2(u_2)) \times H_2(x - u_2) & (x \geq u_2) \end{cases}$$

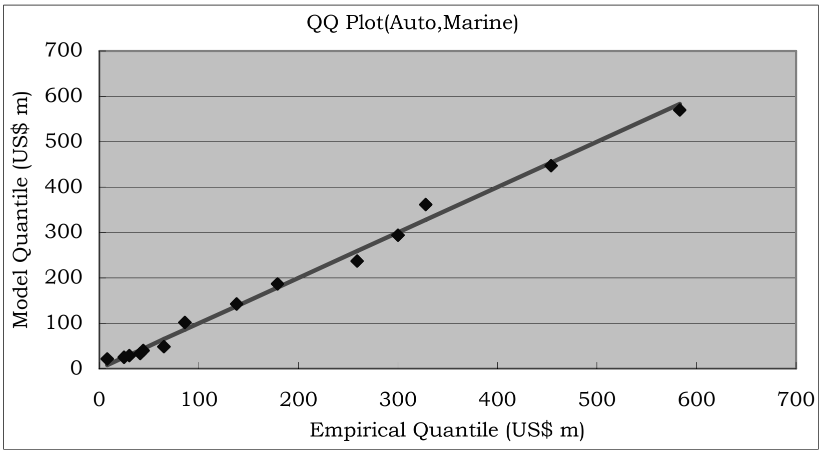
$$G_2(x) = 1 - \left(\frac{\beta_2}{x}\right)^{\alpha_2}, H_2(x) = 1 - \left(1 + \xi_2 \frac{x}{\sigma_2}\right)^{-1/\xi_2}$$

$$u_2 = 65,\ \alpha_2 = 0.714413,\ \beta_2 = 237.347,$$

$$\xi_2 = -0.431108, \sigma_2 = 3746.025$$

The following chart is the Q-Q plot of the severity distribution adopted.

(Q-Q plot)



c. Risk Aggregation

Fire and other lines' severity distributions are aggregated with Gumbel copula.

$$F(x_1,x_2)=C(F_1(x_1),F_2(x_2))=\exp\left\{-\left[\left(-\log F_1(x_1)\right)^{\alpha}+\left(-\log F_2(x_2)\right)^{\alpha}\right]^{1/\alpha}\right\}$$

$$C(t_1,t_2)=\exp\left\{-\left[\left(-\log t_1\right)^{\alpha}+\left(-\log t_2\right)^{\alpha}\right]^{1/\alpha}\right\}$$

Parameter α made $\alpha=\dfrac{1}{1-\tau}$ = 1.895833 making use of Kendall's τ =0.472527 which was requested from observation.

(4) Result

Following the model above, we calculate the aggregate loss distribution by the Monte Carlo simulation. The result is as following table. For comparison purpose, additional two results are provided,

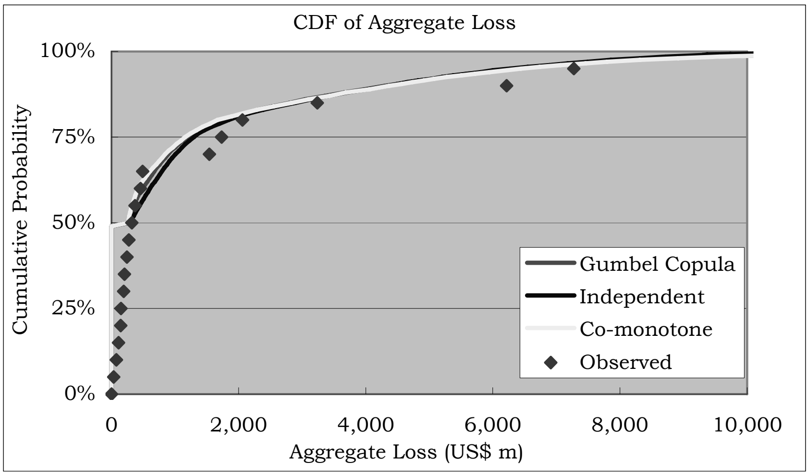
one - "fire" and "other lines" are regarded independent
two - the two items are regarded co-monotone.

Prob.	Return Period	Fire	Other Lines	Total		
				Gumbel Copula	Independent	Co-monotone
%ile	year					
70.0%	3.33	700	144	893	1,007	844
75.0%	4.00	943	215	1,189	1,296	1,158
80.0%	5.00	1,378	290	1,721	1,844	1,668
85.0%	6.67	2,446	382	2,801	2,887	2,828
90.0%	10.00	3,873	500	4,307	4,355	4,373
95.0%	20.00	5,785	657	6,375	6,266	6,442
95.5%	22.22	6,056	682	6,654	6,549	6,738
96.0%	25.00	6,359	706	6,936	6,890	7,066
96.5%	28.57	6,717	739	7,302	7,180	7,456
97.0%	33.33	7,032	769	7,685	7,498	7,801
97.5%	40.00	7,498	819	8,183	7,860	8,317
98.0%	50.00	8,049	867	8,757	8,318	8,916
98.5%	66.67	8,641	923	9,440	8,714	9,564
99.0%	100.00	9,522	1,014	10,473	9,303	10,537
99.5%	200.00	11,017	1,156	12,003	10,333	12,173
99.9%	1000.00	13,648	1,430	15,108	13,482	15,078
Mean		1,075	144	1,218	1,218	1,218
Standard deviation		2,070	238	2,272	2,147	2,130

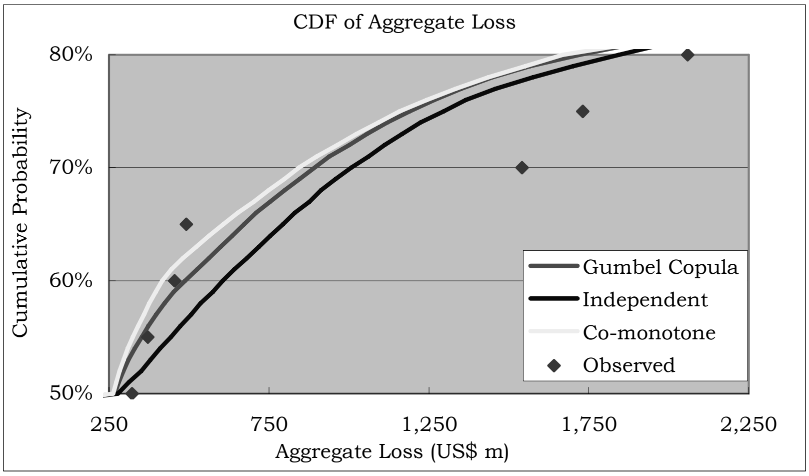
The result based on Gumbel copula is closer to that of co-monotone assumption than that of independent. This tendency is more remarkable in the right tail of the distribution. For example, 99th percentile value of co-monotone assumption is nearly equal to that of Gumbel copula, while the result of the independent assumption shows 11.2% lower than that.

In the Japanese solvency standard, the catastrophe risk of whole company is the sum of risks of each line. It means that risks of each line are assumed to be co-monotone. The assumption is supported by this example.

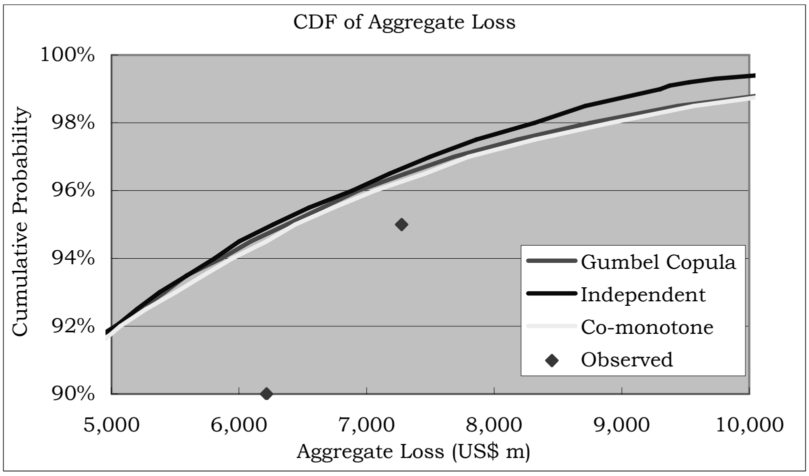
(Cumulative Distribution Function)



(Cumulative Distribution Function: Expanding the part of cumulative probability 50% to 80%)



(Cumulative Distribution Function: Expanding the part of cumulative probability 90% or more)



5 Utilization example of the natural disaster model in Japan

In April 2005 legal reserve system and solvency regulation was revised.

5-1 Unearned premium

It was assumed that frequency and severity of the large-scale, natural catastrophic risk were quantitatively measured, and an insurance company must set aside the expected value as liability reserves.

$$U = \frac{R+E}{P}$$

U : Unearned premium

R : Large-scale natural catastrophic disaster fund (expectation of the payment of the particular business year that is estimated rationally with the risk model that satisfies fixed requirements as the premium that corresponds to large-scale natural catastrophic risk). The part that is covered by reinsurance is deducted.

E : Earned premium other than R
Recent expenses plus recent average payment (occurrence basis) other than R
When amount of the expense or amount of the payment is temporarily high because observation period is short, other rational method is admitted.

P : Premium adjusted by assumed interest that corresponds in the particular business year and is calculated with earned premium as the foundation.
assumed interest = unearned premium
× assumed interest rate / (1 + assumed interest rate)

Requirements for the risk model that can be used for the calculation of large-scale natural catastrophic fund

(1) When there is an engineering accident occurrence model that satisfies the requirements below.

- As for all the assumed insurable contingencies, occurrence place and strength, etc. must be evaluated stochastically with the probability theory based on an engineering theory.
- The phenomenon generated by the insurable contingency is evaluated on the basis of engineering theory.
- The relation of the phenomenon occurred by the insurable contingencies with the weakness estimated on the basis; strength, usage and other element of the insured must be evaluated with an engineering theory.
- Terms and conditions of insurance payment are considered.

(3) When there is no engineering accident occurrence model, the theoretical distribution accident occurrence model that satisfies the requirements below.

- The data observed for a long term as historical earnings at identical terms must be used.
- The historical data used must be the one modified the price level, the content of the coverage, and the condition of risk accumulation.
- The relation of the phenomenon occurred by the insurable contingencies with the weakness estimated on the basis; strength, usage and other element of the insured must be evaluated with an engineering theory.
- Terms and conditions of insurance payment are considered.
- Catastrophic risk must be evaluated with an engineering method or some appropriate method.

5-2 Catastrophe loss reserve

The amount of the insurance payment of the natural disaster of return period 70 year scale whose scale equals to the scale of Typhoon Isewan (1959) is the revised upper limit amount of the catastrophic loss reserve. When the reserving amount of accumulation is less than that, an insurance company must set the reserving plan of the catastrophic loss reserve and proceed to reserve it according to the plan.

(1) Minimum limit amount

An insurance company must save the amount more than the fixed minimum limit amount every year. The minimum limit amount is the expectation of the premium for the large-scale natural catastrophic risk per year. However, when the calculation of this amount is difficult, the amount that 50/100 is multiplied by the large-scale, natural catastrophic fund can be used alternatively.

(2) Reserving maximum limit amount

The amount that is not less than the estimated net payment in case of a large-scale catastrophic disaster. The excess probability at the amount in the risk curve is 1.4 percent point (70 years in return period).

(3) Revision of solvency margin standard

(a) Solvency margin ratio
In addition to the reserves to cover claims payments and payments for maturity-refund of savings type insurance policies. Etc., it is necessary for general insurance companies to maintain sufficient solvency in order to provide against risks that may exceed their usual estimates. The solvency margin ratio means the ratio of “solvency margin of general insurance companies by means of their capital, reserves, etc.” to “risks that will exceed their usual estimate”, as calculated as below.

$$\text{Solvency Margin Ratio (\%)} = \frac{\text{Solvency Margin, i.e. the total amount of accumulations such as capital (fund), reserves, etc. prepared for risks that exceed usual estimates}}{\text{Total of risks that exceeds usual estimates} \times 1/2} \times 100$$

The risks mentioned in the denominator are insurance risk, assumed interest rate risk, asset management risk and business administration risk. Moreover there are two types of insurance risk. One is general insurance risk and the other is catastrophic risk. Catastrophic risk is the risk of loss caused by natural catastrophes such as earthquake, storm, flood, etc.

The solvency margin ratio is one of the indices that the supervisory authority utilizes in order to judge the management soundness of a general insurance company. It is understood that problems concerning the management soundness of a general insurance company will not arise if the ratio is 200% or more.

(b) Modification of calculation method of Catastrophe risk

It is provided that the catastrophic risk quantity by wind and flood is the amount which corresponds to the damage by wind and flood of return period 70 year scale at the time of the calculation of the solvency margin ratio. In addition an insurance company is required to keep the solvency which corresponds to this.

The catastrophic risk quantity used be the bigger one of the amount of Kanto large earthquake scale risk and Typhoon Mireille (1991 No.19) scale risk. In 2005 the wind and flood disaster level for catastrophe risk evaluation was revised to much higher, from Typhoon Mireille to Typhoon Isewan (1959) whose return period is 70 years.

The above modification makes it clear that a non-life insurance company in Japan required to manage catastrophic risk sufficiently and at least built the structure that is solvent enough to the natural catastrophic disaster of return period 70 year scale typhoon.

6 Other issues - Secondary uncertainty

Secondary uncertainty is every uncertainty that originates in the process of the modeling. It can be divided into the model risk and the parameter risk. In the report of IAA Insurer Solvency Assessment Working Party, "A Global Framework for Insurer Solvency Assessment", the uncertainty risk is defined as "the risk that the models used to estimate the claims or other relevant processes are misspecified or that the parameters within the models are misestimated".

Since modeling is simplification of the actual world, the model risk is inevitable to a degree. Especially, as for the natural catastrophic risk model, it deals with the event which rarely occurs or which has not yet experienced. Thus the influence of the secondary uncertainty is more serious because the data that is available to identify the model and to estimate the parameters is limited.

To compare the result of more than one engineering risk model or that of an engineering risk model and a statistical risk model is effective in examining the influence of the model risk. In addition, the underestimate of the risk is more serious than the overestimate for the purpose of pricing and risk management. Therefore, we can mitigate the influence of secondary uncertainty by estimating the parameters with safety loading.

It is not necessary to include safety loading to all the parameters. It is sufficient to consider the parameters whose influence is large. The influence of the parameter risk of each parameter is analyzed by sensitivity test. Or the parameter risk is sometimes embedded into model as a distribution instead of safety loading.

Simple calculation is shown below.

Suppose X is the intensity of hazard (*e.g.* seismic intensity, wind speed) . $\log X$ follows standard normal distribution. Loss factor Y % (amount of damage ÷ insurable value) is the function of X , namely vulnerability curve is presumed $Y = X^{0.8}$. In considering the uncertainty of vulnerability curve, we compared the result of the following three approaches.

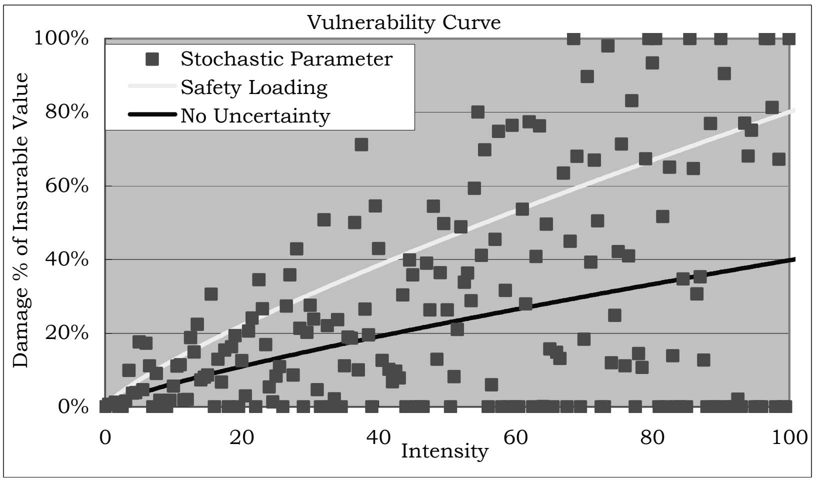
A. (Stochastic Parameter)
Y follows the normal distribution with mean $X^{0.8}$ and standard deviation $1.5X^{0.8}$.

B. (Safety Loading)
Under the same hypothesis as above, Y is determined at the level where true loss factor does not exceed Y with 75% probability.
Namely

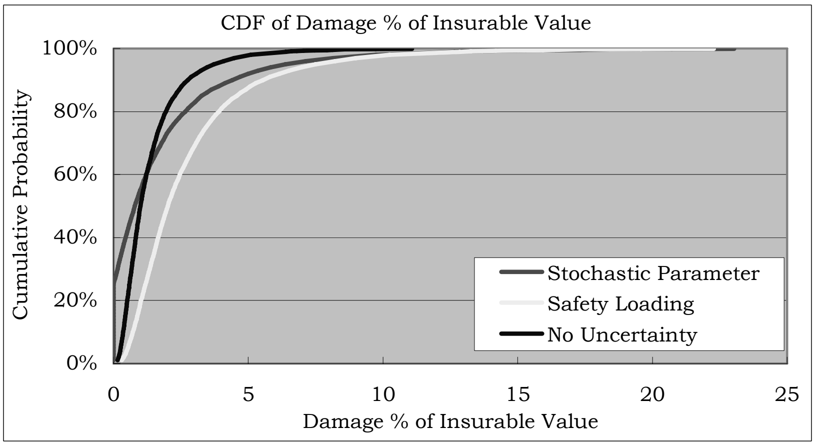
$$Y = \max(X^{0.8} + 0.6745 \times 1.5X^{0.8}, 100)$$

C. (No uncertainty)
Uncertainty is not considered.

As a result, vulnerability curve becomes as following chart.



The result of calculating the distribution of loss factor using these vulnerability curves is the following chart.



Loss factor B (using safety loading) exceeds others at all quantile. On the other hand, the magnitude relation of the cumulative probability between Loss factor A (Stochastic Parameter) and Loss factor C (No uncertainty) is different between the right tail of the distributions and left tail. For example, at the 99th percentile, loss factor A, B and C is 12.6%, 12.8% and 6.4% respectively.

If it is allowed for uncertainty by adding safety loading, 99th percentile itself becomes a random variable. Therefore some judgment has to be made concerning which '99th percentile' should be chosen (mean, 75th percentile, etc.). On the other hand, in the case that parameter is designated as random variable, that meaning is clear.

In addition, if risk is measured in terms of the difference between a certain quantile and the expected loss, overestimating the loss amount by safety loading does not mean conservative quantification of risk.

The following table shows the risk measured in terms of 99th percentile minus expected loss.

Damege % of Insurable Value

	Stochastic Parameter	Safety Loading	No Uncertainty
Mean	1.7	2.8	1.4
99%ile	12.6	12.8	6.4
Difference	11.0	10.1	5.0

In the case using safety loading, both average and 99th percentile are higher than other cases. Nevertheless, the risk (=99th percentile – mean) is largest in the case using stochastic parameter, not safety loading. The example implies that secondary uncertainty should be embedded in the model as a distribution from the viewpoint of conservativeness.

7 Summary

In Japan, where a great number of natural disasters, such as typhoons, hailstorms and earthquakes, occur every year, it is a critical issue how to evaluate natural catastrophic risk and form reinsurance arrangements. Recently, demand for coverage against natural disasters has been more and more increasing. In cases in which reinsurance arrangement cannot be sufficiently achieved, various countermeasures have been taken effect, such as insurance securitization and insurance derivatives.

The supervising authorities requested a certain level of regulation where non-life insurance companies do not fall into insolvency due to natural disasters, and, consequently, insurance business laws were revised. There, modification of legal reserve calculation method for natural disasters and solvency margin ratio is to be executed in the 2005 accounting period.

The security level regulated in the law does not necessarily guarantee the safety of insurance companies. Insurance companies, in their own decision, synthetically measure and evaluate all the risk surrounding them, such as, not only natural disasters, but also insurance risk and investment risk. Thus, they are managing their risk in order to secure their financial soundness. As we have examined so far, such security level now has a significant meaning in Japan that insurance companies should prepare for the wind and flood disasters of return period 70-years.

In future, it is expected that more enormous natural disasters may occur and safety barometers may change. In Japan, a country of natural disasters, insurance companies will play a more and more important role and we must further promote research and development of risk evaluation method.

References

[1] KLUGMAN, S. A., H. H. PANJER and G. E. WILLMOT (1998), *Loss Models: From Data to Decisions,* JohnWiley & Sons, New York.

[2] SANDERS, D. E. A. (2005), The Modeling of Extreme Events.

[3] YAMAI, Y., and T. YOSHIBA (2002), Comparative Analyses of Expected Shortfall and Value-at-Risk under Market Stress.

[4] General Insurance (The Institute of Actuaries of Japan)

[5] White Paper on Disaster Management (Cabinet Office, Government of Japan)

YUKIE UCHIYAMA: Nissay Dowa General Insurance Company Limited, Accounting Dept.
St. Lukes Tower, 8-1, Akashi-cho, Tyuou-ku, Tokyo, Japan
Phone: 81-3-5550-0232 Fax: 81-3-5550-0310
E-mail: yukie.uchiyama@nissaydowa.co.jp

SHIGEO WATANABE: Nissay Dowa General Insurance Company Limited, Fire and Casualty Dept.
St. Lukes Tower, 8-1, Akashi-cho, Tyuou-ku, Tokyo, Japan
Phone: 81-3-5550-0222 Fax: 81-3-5550-0313
E-mail: shigeo.watanabe@nissaydowa.co.jp