COMPARISON FOR ESTIMATION RISK OF O/S AND IBNR RESERVE USING THE CHAIN LADDER METHOD, THE CREDIBILITY MODEL AND MCMC METHODS

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Abstract. The simulations and comparisons between credibility models and several MCMC methods show that the fully Bayesian approach using MCMC method produce a smaller estimation errors and the ratio of standard error to estimated reserves than that of credibility model, which lead good results for the estimated reserves.

1. Introduction

For MCMC (Markov Chain Mote Carlao) simulations, the simple accessibility and wide applicability of the sampling based approach to Bayesian inference suggests the feasibility of a common purpose software for Bayesian analysis. Up to this view point, the user simply writes a short lines Gibbs or Metropolis-Hastings sampler code for the problem at hand, and modifies it to fit whatever subsequent problems come along. Users have also used that some high-standard languages (*S-Plus, XLISP-STAT*) which are convenient for the data entry, graphical convergence monitoring, and posterior summary statistics, while lower-level compiled languages(*C* or *Fortran*) are needed to facilitate the enormous amount of random generation and looping in the sampling procedure.

The lower-level language is the more difficult one, since the computer has to be coded to understand a statistical model, or the prior and likelihood components, and make the necessary sampling distributions before sampling. These problems to overcome, a program has been developed and available via internet as freeware, i.e. BUGS (Bayesian inference Using Gibbs Sampling). This program is written by *S-Plus*-like syntax for specifying hierarchical model and developed by MRC Biostatistics Unit at the University of Cambridge.

The program determines the full conditional distributions necessary for the Gibbs sampler and the non-explicit conditional distributions for the Metropolis-Hastings sampler by converting this syntax into an acyclic graph, the node of which correspond to the data and parameters in the model. Bugs successively samples from the parameter nodes, writing the output to a file for subsequent convergence assessment and posterior information. We estimate our model by Gibbs sampling and Metropolis-Hastings sampling using the Bugs software. In the description of Gibbs sampling, four procedures are required to implement a Gibbs sampling:

1.1 Starting values must be provided for all unobserved nodes (parameters any missing data):

In principle, the choice of the starting values is not important since Gibbs sampler and other MCMC methods should be run long enough for it to ignore its initial values. However very extreme starting values could cause to a long

"Burn-in" to sample. In unfortunate case, the sampler may fail to converge towards the posterior distribution, this possibility being aggravated by numerical instability in the extreme tails of the posterior.

1.2 Full conditional distributions for each unobserved node must be constructed and methods for sampling from them decided upon:

Gibbs sampling works by iteratively getting samples from the full conditional distribution for a node is the distribution of that node given current or known values for all other nodes. The full conditional distribution for the precision parameters can be easily worked out.

A particularly useful application of the Metropolis-Hastings sampler is where an intractable density arises within a Gibbs sampler as the product of a standard density and another density, e.g. $\pi(x) \propto \varphi(x) \cdot \phi(x)$, where $\phi(x)$ is a standard density that can be sampled. The general prescription shows us that the full conditional is proportional to the product of the prior, which can be taken from data, experience or aggregated information of industry and the likelihood terms.

1.3 The output must be monitored to decide on the length of the "Burn-in" and the total run length, or perhaps to identify whether a more computationally efficient parameterization or other MCMC algorithm is required.

The values for the unknown quantities generated by the Gibbs sampler can be graphically and statistically summarized to check.

1.4 Summary statistics for quantities of interest must be calculated from the output, for inference about the true value of the unobserved nodes.

1.5 Standard deviation in summary statistics shows the degree of fluctuation of simulations.

2. Data of Mark (1996): Comparison among the Chain Ladder method, the De Vylder-Mack Credibility model and MCMC method using Gibbs sampler

[Reserve estimating procedure]

The overall approach to a reserve valuation problem can be broken into four phases.

2.1 Review of the data to identify its key characteristics and possible bias. Balancing of the data to other verified sources should be undertaken at this point.

2.2 Application of appropriate reserve estimation methods and selection of hyperparameters and its evaluation.

2.3 Evaluation of the conflicting results of the various reserve methods used, with an attempt to reconcile or explain the bases for different projections. At this point the proposed reserves are evaluated in contexts outside their original frame of analysis.

2.4 Prepare projections of reserve development that can be monitored over the subsequent calendar periods. Deviation of actual from projected developments of counts or amounts is one of the most useful diagnostic tools in evaluating accuracy of reserve estimation: The typical measure of error is *mean square error* (*MSE*)

$$M.S.E. = \sum_{i=1}^{n} (Z_i - \hat{Z}_i)^2 / n ,$$

where Z_i is actual claim amount and $\hat{Z}_i$ is estimated claim amount.

2.5 The size of mean square error and the ratio of mean square error to estimated reserve can be index of "better fitted model".

The comparison among the Chain-Ladder, the credibility model and the MCMC method lies on different approaches. We introduce first the Chain Ladder method for the comparison of the Chain-Ladder method with credibility model. The second is that the basic model is equivalent to each other to compare between the credibility model the MCMC method. This means that the development effect can be directly comparable with each other.
The Chain-ladder method produces somewhat different delay effect f_j, which is not comparable with that of credibility model and the MCMC methods. In the credibility model, for instance, the development effect y_k can be directly obtained by the data if the expected value $E[X_{ik} \mid \theta_i] = p_i \mu(\theta_i) y_k$ is assumed (see section 3.3 in this thesis and Mack: 1996, p235). The $\mu(\theta_i)$ plays an important role in the credibility model however $\mu(\theta_i)$ itself is not so well defined in the credibility model. The MCMC methods produce the delay effect and contain itself the procedure of the estimation of delay effect.

The classical Chain Ladder method produces the final estimated amount, which can't be comparable with estimated values by other models. But the stochastic Chain Ladder method by Mack (1993, 1994) can produce the standard error of the estimates, which help compare with other models. The original data set is given in the appendix.

A. The Chain Ladder method:
[Model and assumption]

We assume that a run-off triangle is given, filled with cumulative loss figures X_{ij}, i.e. X_{ij} is the total amount paid in year of origin *i* and in the following *j* years, on behalf of losses incurred in year of origin *i*.. The Chain Ladder method is a procedure to complete this triangle to a square, or eventually to estimate value $X_{i\infty}$, once $X_{0\infty}$ is given. The basic assumption is that the columns in the triangle are proportional, apart from random fluctuations. So if $X_{13} : X_{14}$ = 2 : 3, we must assume $X_{23} : X_{24}$ = $X_{33} : X_{34}$ = $X_{43} : X_{44}$ = ...= 2 : 3, approximately.

The assumptions imply that the run-off pattern over the development years is stable. So the method breaks down if the internal or external impacts cause a change in the run-off pattern.
Now we introduce the stochastic Chain Ladder model. Let $C_{iJ} = X_{i1} + ... + X_{iJ}$ in the multiplicative form $C_{iJ} = C_{i1}F_{i1}F_{i2} \cdots F_{i,J-1}$, where $F_{ij} = C_{i,j+1} / C_{ij}$. Random variable F_{ij} is independent of accident years i and its expected value can be expressed by $E[F_{ij}] = f_j, 1 \le i \le I,\ 1 \le j \le I-1$.

1. $E[C_{i,j+1} \mid C_{i1},......,C_{ij}] = C_{ij} f_j$,
2. Independence of accident years,
3. $Var[C_{i,j+1} \mid C_{i1},......,C_{ij}] = C_{ij}\sigma_j^2$,

where f_j, σ_j^2 are estimated by Mack(1996) as follows:

$$\hat{f}_j = \sum_{i=1}^{I-j} C_{ij} /(C_{1j} + .. + C_{I-j,j}) \cdot C_{i,j+1} / C_{ij} ,$$

i.e. $\hat{f}_1;......;\hat{f}_5$; 1.59; 1.49; 1.18; 1.07; 1.05, respectively for this data and

$$\hat{\sigma}_j^2 = 1/(I-j-1) \cdot \sum_{k=1}^{I-j} C_{kj} (C_{k,j+1} / C_{kj} - \hat{f}_j)^2 ,$$

i.e. $\hat{\sigma}_1;......;\hat{\sigma}_5$; 12.95; 9.07; 7.03; 3.78; 2.03, respectively for this data, $1 \le j \le J-2$.

For *j*=*i*-1, Mack uses log linear regression to estimate

$$\hat{\sigma}_{I-1}^2 = \min(\hat{\sigma}_{I-2}^4, \min(\hat{\sigma}_{I-3}^2, \hat{\sigma}_{I-2}^2)) .$$

Had $f_{I-1} = 1$, then we could have put σ_{I-1}^2 = 0.
Finally, the standard error, i.e. the square root of the mean square error consists of the statistical random error and estimation error:

$$MSE(\hat{R}_i) = E((R_i - \hat{R}_i)^2 \mid D) = E((C_{iI} - \hat{C}_{iI})^2 \mid D) = MSE(C_{iI}) ,$$

where $R_i = C_{iI} - C_{i,I+1-i}$

$\hat{R}_i = C_{i,I+1-i}(\hat{f}_{I+1-i} \cdot .... \cdot \hat{f}_{I-1} - 1)$, and

$D = \{C_{ij} \mid i + j \le I + 1\}$.

MSE can be divided by two parts:

$$MSE(\hat{R}_i) = MSE(\hat{C}_{iI}) = Var(C_{iI} \mid D) + (E(C_{iI} \mid D) - \hat{C}_{iI})^2 .$$

The following formulae are given for the mean square error of

$$MSE(\hat{R}_i) = \hat{C}_{iI} \sum_{j=I+1}^{I-1} \frac{\hat{\sigma}_j^2}{\hat{f}_j^2} \left(\frac{1}{\hat{C}_{ij}} + \frac{1}{\sum_{k=1}^{I-j} C_{kj}} \right) \text{ and}$$

$$MSE(\hat{R}) = \sum_{i=2}^{I} \left\{ (s.e.(\hat{R}_i)^2 + \hat{C}_{iI} (\sum_{k=i+1}^{I} \hat{C}_{kI}) \sum_{j=I+1-i}^{I-1} \frac{2\hat{\sigma}_j^2}{\hat{f}_j^2} \left(\frac{1}{\sum_{k=1}^{I-j} C_{kj}} \right) \right\}.$$

The first two assumptions seem intuitively reasonable, and it is possible to demonstrate that they are consistent with chain ladder method more formally. The third assumption follows from the calculation of f_j. A corollary of assumption 3 is that the development factor is not correlated.

So, for example, if the development factor is high in one period, it dose not follow that it should be not high (or low) in the next period. This will not always be reasonable: for example, if a company decides to change its claims handling procedure so that a large number of outstanding claims are settled in one development period, it is likely that the following period will have a low development case.

The estimate of the Chain Ladder method, e.g. delay effect f_j can not be directly compared to that of the credibility model and MCMC method because the estimates in the Chain Ladder model is not comparable to that of the other models, i.e. the Chain-Ladder produces the delay effect f_j under different basis. But we can compare with the standard errors and the ratio of standard error to estimated reserve, which are produced by the Chain-Ladder, the credibility model and MCMC method.

B. Credibility model:
[Model and Assumptions]

1. For each accident year $X_{i1},......,X_{iI}$ depend on unknown parameter θ_i, which is a variable for accident year factor.
2. In each accident year *i*, $X_{i1},......,X_{iI}$ are independent.
3. The accident year vector (*X*, θ) are independent i.e. independent of accident years.
4. The θ_i`s are identically distributed.
5. The multiplicative assumption of development effect y_k and risk parameter θ are given as follows:

$E[X_{ik} \mid \theta_i] = p_i \mu(\theta_i) y_k$, where y_k is development effect and p_i is premium volume.

6. The variance assumption is given as follows:

$$Var[X_{ik} \mid \theta_i] = p_i \sigma^2(\theta_i) y_k^r \text{, where } r\text{=1 or 2.}$$

We define the structure parameters

$$m = E[\mu(\theta_i)],\ u = E[\sigma^2(\theta_i)],\ w = Var[\mu(\theta_i)],$$

$$E^*(Z_{ij} \mid \theta_i) = c_i Z_{i+} + (1-c_i)m, \quad j > I+1-i \text{ with } Z_{i+} = \sum_{k=1}^{I+1-i} q_{ik} Z_{ik} / q_{i+} ,$$

$$c_i = q_{i+} / (q_{i+} + u/w) \text{, where } q_{ik} = p_i y_k^{2-r} .$$

The mean square error of estimation of R_i is calculated by development triangle

$$D = \{X_{ik} \mid i+k \le I+1\},$$

$$MSE(\hat{R}_i) = E[(R_i - \hat{R}_i)^2 \mid D] = Var(R_i \mid D) + (E(R_i \mid D) - \hat{R}_i)^2$$

$$\cong p_i u(y_{I+2-i}^r + ... + y_I^r) + v_i^2 (1-c_i) w (y_{I+2-i} + ... + y_I)^2$$
$$+ v_i^2 (E^*(Z_{ij} \mid \theta_i))^2 \sum_{j,k} Cov(\hat{y}_j, \hat{y}_k)$$

with $I+2-i \le j,\ k \le I$ and

$$Cov(\hat{y}_j, \hat{y}_k) = \frac{\delta_{jk}(v_1 + ... + v_{I+1-k}) y_k^r u + (v_1^2 + ... + v_{I+1-k}^2) y_j y_k w}{(p_1 + ... + p_{I+1-j})(p_1 + ... + p_{I+1-k})} .$$

This model was discussed in chapter 3 and the mean square of estimation of R_i is produced by Mack(1996). The assumption 5, claim payment can be divided into $E[X_{ik} \mid \theta_i] = E[N_{ik} \mid \theta_i] \cdot E[Y_{ik} \mid \theta_i]$, with N_{ik} is number of payments and Y_{ik} is average claim amount, where the premium volume is hidden in $E[N_{ik} \mid \theta_{ik}]$. We use r =1 for this model, and there is no significant difference between r =1 or 2 for this data by trials.

C. Fully Bayesian approach using Gibbs sampler:

The fully Bayesian approaches can be expressed as follows in comparison to credibility model. The assumptions are ascribed by the model We assume that all Y_{ij} conditionally independent and that Y_{ij} ~ *Normal*(θ_i , *ss*) for all *i* and *j*. The parameter θ_i denote the delay effect. Given *a* and *cc*, the θ_i are assume to be conditionally with θ_i ~Normal(a, cc) for all *i*. We complete this model by letting *a*~gamma(#1, #2), *cc*~gamma(#3, #4) and *ss*~gamma(#5, #6). The next Normal-Gamma model is performed by the same way. The selection of hyperparameters are discussed later.

1. Normal-Normal

Y[i, j] ~ norm(mu[i,j], ss)
mu[i, j] = theta[i]*P[j]*X[j]
theta[i] ~ norm(a, cc)

a ~ gamma(#1 , #2)
cc ~ gamma(#3 , #4)
ss ~ gamma(#5 , #6)

where theta[i] is development effect, P[j] is premium volume, X[j] denotes inflation effect and #1,#2,#3,#4,#5,#6 denotes *a prior*.

2. Normal-Gamma

Y[i, j] ~ norm(mu[i,j], ss)
mu[i, j] = theta[i]*P[j]*X[j]
theta[i] ~ gamma(a, cc)

a ~ gamma(#1 , #2)
cc ~ gamma(#3 , #4)
ss ~ gamma(#5 , #6)

where theta[i] is development effect, P[j] is premium volume, X[j] denotes inflation effect and #1,#2,#3,#4,#5,#6 denotes *a prior*.

The check of the results by selection of hyperparameters and calculation of estimates are discussed as follows: In the MCMC methods we need the information about parameter of *a prior* distribution. As mentioned in the section 2.4 in variance component model, the information about parameter of *a prior* distribution can be obtained from the variance components by the credibility model.

For instance, in the Normal-Normal model *a, cc* and *ss* can be interpreted as the average of development effect, the variance of development effect and total variance according to accident years respectively. These *a* and *ss* correspond to $E[y_k]$ and $u = E[\sigma^2(\theta_i)]$, respectively in the previous credibility model. Therefore the variance components by credibility model or non-informative approach in Bayesian analysis by Jepperys(1961)give us the information about *a prior*. The non-informative analysis in Bayesian approach without specific a prior takes more time than with specific a prior information.

We first check the results by variations of hyperparameters in the model Normal-Normal:

a. *a*~(0.15, 1), *cc*~(1000, 1), *ss*~(1000, 0.1): This a prior is used in the next section for detail analysis and comparison among other models.
   Estimated IBNR reserve: 22,803
   Standard error: 4,346
   Estimated value $\hat{a}$=0.1421,

Estimated value $\hat{cc}$ =976.8,

Estimated value $\hat{ss}$ =1.042E-4.

b. a~(2, 1), cc~(500, 1), ss~(500, 0.1)

Estimated IBNR reserve: 22,824
Standard error: 6,152
Estimated value $\hat{a}$ =0.1458,

Estimated value $\hat{cc}$ =489.7,

Estimated value $\hat{ss}$ =5.243E-5.

c. a~(1, 1), cc~(100, 1), ss~(100, 0.1)

Estimated IBNR reserve: 22,792
Standard error: 13,440
Estimated value $\hat{a}$ =0.1423,

Estimated value $\hat{cc}$ =100.0,

Estimated value $\hat{ss}$ =1.109E-5.

The results show us that the variations of hyperparameters realize a little difference in the dispersion of the estimated IBNS reserve. But the standard error is growing from 4,346 to 13, 440 rapidly. So the standard error is a measure of the selection of the models. Then we can choose parameter of a prior distribution using standard error. Another measure of section can be considered e.g. the ratio of standard error to estimated reserves. An argument about the distribution of *a prior* arises.

Whether a prior has normal distribution or gamma distribution, we don't have such previous information. This argument is related with the degree of dispersion of density for *a prior*. Therefore first we have to simulate the models and then the standard errors can be compared with each other. The small standard errors of estimates have high probability against choosing the "bad fitted models". We have another measure of "better fitted model", i.e. the ratio of standard error to estimated reserves, which means the coefficient of variation in the statistical context. We use this measure also for the comparison and analysis.

2.1. The comparison of procedure for the calculation

First of all we have to check the assumptions for several models. The differences among models have to be detected how we can compare with each other directly or not. Although mathematical basis or some assumptions are not equivalent, i.e. the delay effect cannot compared directly, we can compare with the estimated reserves, standard error and the ratio of standard error to estimated reserves.
The criterions for a better fitted model can be standard error and the ratio of standard error to estimated reserves. If new candidate model appears, then we can detect this procedure again and select the model.

The Chain-Ladder method cannot be compared directly with credibility model or MCMC methods, although it produces the delay effect f_j and σ_j^2 , i.e. $\hat{f}_j = \sum_{i=1}^{I-j} C_{ij} /(C_{1j} + .. + C_{I-j,j}) \cdot C_{i,j+1} / C_{ij}$, $\hat{f}_1$;......; $\hat{f}_5$;1.59; 1.49; 1.18; 1.07; 1.05, respectively and $\hat{\sigma}_j^2 = 1/(I-j-1) \cdot \sum_{k=1}^{I-j} C_{kj} (C_{k,j+1} / C_{kj} - \hat{f}_j)^2$, $\hat{\sigma}_1$;......;$\hat{\sigma}_5$; 12.95; 9.07; 7.03; 3.78; 2.03 for this data. Because the basis of assumption for f_j is different from that of credibility model and MCMC methods.

Between credibility model and MCMC methods we can compare the delay effect each other, because these mathematical form are equivalent to each other, but the estimation methods are different. Although the delay effect cannot be compared with all methods, we can compare the estimated reserves, the standard error of estimates and the ratio of standard error to the estimated reserves, which is another measure of quality for the models. In the MCMC method the delay effect **theta[i]** can be produced by the model. We compare the credibility model and MCMC method in the delay effect(see **[Figure 2.1]**).

The Chain-Ladder method cannot be compared with other models in the delay effect. Although it produce the delay effect f_j, it is not comparable with that of credibility model and MCMC methods directly. Because the basis of delay effect is different from each other. The data was modified by deflation factor in the original data set. Then we do not need to insert the inflation effect for the estimation. The values $\mu(\theta_i)$, which are assumed to be 1 in the credibility model by Mack(1996, p.236), play a main role in the credibility model. The MCMC method produces the delay effect and contains itself the procedure of the estimation of delay effect. In the MCMC method, we need only to calculate the fully Bayesian estimator using a prior information. The information for a prior in the MCMC method can be obtained either by the credibility model or by the non-informative analysis in Bayesian approach directly.

Figure 2.1. Comparison of the development effect

Coefffi.	Credibility model	Coeffi.	Normal-Normal	Normal-Gamma
y(1)	0.297	theta(1)	0.2977	0.2955
y(2)	0.178	theta(2)	0.1792	0.1768
y(3)	0.201	theta(3)	0.1979	0.1943
y(4)	0.115	theta(4)	0.1119	0.1106
y(5)	0.058	theta(5)	0.0221	0.0458
y(6)	0.049	theta(6)	0.0541	0.0710

2.2. The comparison of results for the calculations

A. Estimations of the total IBNS amount by several models:

The next figures in the table (See **[Figure 2.2]**, **[Graph 2.2]** and **[Graph 2.3]**) show the differences of the estimated claim reserves and the standard error of estimates.

The MCMC methods do not need to calculate or to assume the ambiguous value $\mu(\theta_i)$ for the IBNS estimation. Without any ambiguous and superficial variable we can directly estimate several the IBNS reserve estimation and select a suitable model. These approaches have some advantages for the selection of models. Normally, the distribution of claim amount can be modeled by Normal-Normal model corresponding to De Vylder-Mack credibility model. But in many cases, the density of claim payment and number of claims could have some other distributions. The MCMC method produces the trace of simulations (see Appendix),i.e. the estimated reserves, standard error and the acceptance rate in the Metropolis-Hastings sampler**.** We have two results of MCMC methods, namely: Normal-Normal model and Normal-Gamma model.

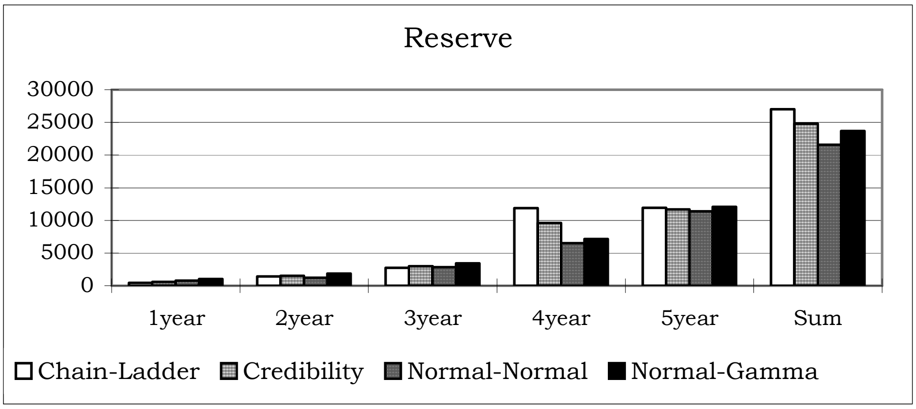
The MCMC models show relative small estimated IBNS reserve and standard errors, and the interval of estimated reserves moves between 22,803 (Norma-Normal model) and 25,573 (Normal-Gamma model), and the interval of standard errors shows 3,982(Normal-Gamma model) and 4,346(Normal-Normal model).

All of the estimated reserve and standard errors by the MCMC models are smaller than that of the Chain Ladder method (the estimated reserve 28,430 and standard error 7,029) and the credibility model (the estimated reserve 26,291 and standard error 5,751). Finally we can conclude that the Normal-Gamma model is a better candidate because this model has moderate estimated reserves, which are neither so extremely larger nor so extremely small with small standard errors. So the ratio of standard error to the estimated reserves can be another measure for selecting of model. We discuss the measure in the next step.

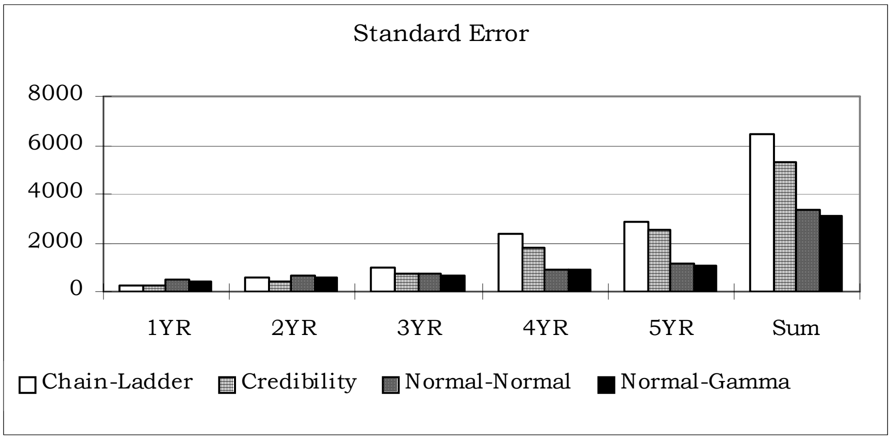
Figure 2.2. Comparison of the estimated reserves and standard errors

	Chain-Ladder	**Credibility model**	**Normal-Normal**	**Normal-Gamma**
Estimated reserves	28,430	26,291	22,803	25,573
Standard Error	7,029	5,751	4,346	3,982

Graph 2.1. Yearly estimated reserves and total estimated reserves



Graph 2.2. Yearly standard error and total standard error



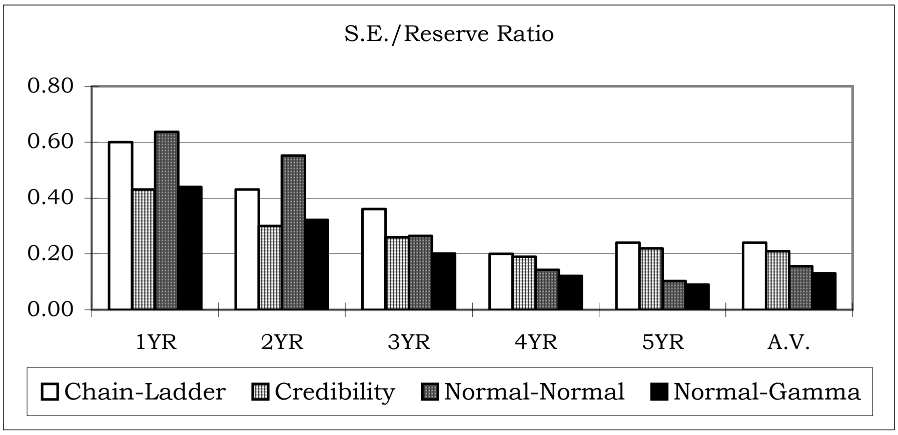
B. Calculation and comparison of standard error (mean square error):

The results show that the Chain Ladder method and the credibility model have the large estimated reserves and standard errors in comparison with MCMC methods (The standard error, i.e. square root of mean square error consists of the statistical random error and estimation error.). The simulations and comparison among the Chain-ladder, credibility model and several MCMC methods show that the fully Bayesian approach using MCMC methods produces smaller standard errors and ratio of standard error to estimated reserve than that of credibility model. We can compare the standard errors, which are produced by MCMC method. If the standard errors of all models are large, we can use the 105% or 110% percentile of standard error loaded loss reserves for conservative account, because MCMC methods can produce the percentile. In all models, we can choose a better candidate for the IBNS estimation. As mentioned, the Normal-Gamma model is a better candidate in the context of the standard error. But the standard error is not only a measure for selecting of model but the ratio of standard error to estimated reserve, i.e. the ratio is 0,19 for Normal-Normal model and 0,16 for Normal-Gamma model (see **[Figure 2.3]** and **[Graph 2.3])**. In this point of view, the Normal-Gamma model is definitely a better candidate model in these models.

Figure 2.3. Yearly standard error and total standard error

	Chain-Ladder	**Credibility model**	**Normal-Normal**	**Normal-Gamma**
1 year	0.60	0.43	0.64	0.44
2 year	0.43	0.30	0.55	0.32
3 year	0.36	0.26	0.26	0.20
4 year	0.20	0.19	0.14	0.12
5 year	0.24	0.22	0.10	0.09
Average	0.25	0.22	0.19	0.16

Graph 2.3. Yearly ratio of standard error to estimated reserves (A.V. means average)



3. Data of Hesselager (1991): Comparison among the Chain-Ladder method, Hesselager-Witting Credibility model and MCMC methods using Gibbs sampler and Metropolis-Hastings sampler

A. Credibility model: Model and assumptions

The analysis of Hesselager is performed by hierarchical credibility approach. We also perform such hierarchical model using MCMC methods. In the credibility model, the delay effect y_k and average claim payment α_k under the assumptions of the expected total claim amount $E[X_{hik} \mid \theta_{hi}] = p_{hi}\mu(\theta_{hi})y_k\alpha_k$ and the expected claim number $E[N_{hik} \mid \theta_{hi}] = p_{hi}\mu(\theta_{hi})y_k$ can be directly obtained by the data, where $\hat{\alpha}_k = \{\sum_{h,i} X_{hik}\}/\{\sum_{h,i} p_{hi} \cdot \widehat{\mu y}_k\}$, $\widehat{\mu y}_k = \sum_{h,i} N_{hik} / \sum_{h,i} p_{hi}$, $\hat{y}_k = \widehat{\mu y}_k / \hat{\mu}$ and $\hat{\mu} = \sum_k \widehat{\mu y}_k$ (see Hesselager:1991, p35) and other assumptions are the same as the previous example. We perform to compare the credibility model with the MCMC methods, i.e. Normal-Gamma, Poisson-Gamma, Poisson-Gamma-Normal model using Metropolis-Hastings sampler and Poisson-Gamma-Gamma model. These models show smaller size of estimation error than that of the credibility model. The ratio of standard error to the estimated reserves can be another measure of better candidate for the models. The data was modified by deflation factor in the data set. Then we do not need to insert the inflation effect for the estimation. The model selection is performed by the previous same procedure. The original data set is given in the appendix. The fully Bayesian approach can be expressed as follows:

B. Fully Bayesian approach using the MCMC methods

The Normal-Gamma model and the Poisson-Gamma model are assumed by the same way of the previous comparison. For the Poisson-Gamma-Gamma model we assume that all Y_{ij} conditionally independent and that Y_{ij} ~ *Poisson*(μ_{ij}) for all *i* and *j*. The parameter θ_i denote the delay effect. Given *a* and *cc*, the θ_i are assume to be conditionally with θ_i ~gamma(η_i , cc) for all *i* given *cc*. The η_i are assumed to be conditionally with η_i ~gamma(*a*, *dd*) given *a* and *dd*. The we complete this model by letting *a*~Normal(#1, #2), *cc*~gamma(#3, #4) and *dd*~gamma(#5, #6). The next Poisson-Gamma-Normal model is performed by the same assumption. It is of interest that the Poisson-Gamma-Normal model is performed by the Metropolis-Hastings sampler because of the non-explicit density.

1. Normal-Gamma model via Gibbs sampler

Y[i, j] ~ normal(mu[i,j],ss)
mu[i, j] = theta[i]*P[j]*X[j]
theta[i] ~ gamma(a, cc)

a ~ gamma(#1 , #2)
cc ~ gamma(#3 , #4)
ss ~ gamma(#5 , #6)

where theta[i] is delay effect, P[j] is premium volume, X[j] denotes inflation effect and #1,#2,#3,#4,#5,#6 denote *a prior*.

2. Poisson-Gamma model via Gibbs sampler

Y[i, j] ~ pios(mu[i,j])
mu[i, j] = theta[i]*P[j]*X[j]
theta[i] ~ gamma(a, cc)

a ~ gamma(#1 , #2)
cc ~ gamma(#3 , #4)

where theta[i] is delay effect, P[j] is premium volume, X[j] denotes inflation effect and #1,#2,#3,#4 denote *a prior*.

3. Poisson-Gamma-Gamma model via Gibbs sampler

Y[i, j] ~ pois(mu[i, j])
mu[i, j] = theta[i]*P[j]*X[j]
theta[i] ~ gamma(etha[i], cc)
etha[i] ~ gamma(a,dd)

a ~ norm(#1, #2)
dd ~gamma(#3, #4)
cc ~ gamma(#5, #6)

where theta[i] is delay effect, P[j] is premium volume, X[j] denotes inflation effect and #1,#2,#3,#4,#5,#6 denote *a prior*.

4. Poisson-Gamma-Normal model via Metropolis-Hastings sampler

Y[i, j] ~ pois(mu[i, j])
mu[i, j] = theta[i]*P[j]*X[j]
theta[i] ~ gamma(etha[i], cc)
etha[i] ~ norm(a,dd)

a ~ norm(#1, #2)
dd ~gamma(#3, #4)
cc ~ gamma(#5, #6)

where theta[i] is delay effect, P[j] is premium volume, X[j] denotes inflation effect and #1,#2,#3,#4,#5,#6 denote *a prior*.

The check of the results by selection of hyperparameters and calculation of estimates are discussed the same method as before. We first check the results by variations of hyperparameters in the model Normal-Gamma model:

a. *a*~(40, 1), *cc*~(1000, 1), *ss*~(1000, 1): This a prior is used in the next section for detail analysis and comparison among other models.
Estimated IBNR reserve: 12,452
Standard error: 2,916
Estimated value $\hat{a}$ =18.86,
Estimated value $\hat{cc}$ =912.1,
Estimated value $\hat{ss}$ =2.187E-4.

b. *a*~(20, 1), *cc*~(500, 1), *ss*~(500, 1)
Estimated IBNR reserve: 11,013
Standard error: 3,345
Estimated value $\hat{a}$ =0.2503,
Estimated value $\hat{cc}$ =500.6,
Estimated value $\hat{ss}$ =1.434E-4.

c. *a*~(10, 1), *cc*~(100, 1), *ss*~(100, 1)
Estimated IBNR reserve: 10,847
Standard error: 7,155,
Estimated value $\hat{a}$ =0.2835,
Estimated value $\hat{cc}$ =101.1,
Estimated value $\hat{ss}$ =3.119E-5.

3.1 The comparison of procedure for the calculations

The results show us that the variations of hyperparameters realize some large difference between the dispersion of the estimated IBNS reserve. The standard errors grow from 1,239 to 2,916 rapidly, whereas the estimated IBNS reserves move from 10,891 to 12,146(See **[Figure 3.1]**). So the standard error and the ratio of standard error to estimated reserves could be measures for the selection of the models because small IBNS reserve and large standard error, i.e. the ratio of standard error to estimated reserves can be less fitted model. Using combination of a prior information under specific assumption for density, we can explore the new models.

The delay effect can be produced by MCMC method and absolute value of delay effect is not equivalent and not comparable to the values of the delay effect in the credibility model directly because of the term i.e. average claim payment α_k. However the relative delay effect can be comparable to delay effect of the credibility model. The coefficients of delay effect of MCMC methods have to be summed and be divided by each coefficient, e.g.

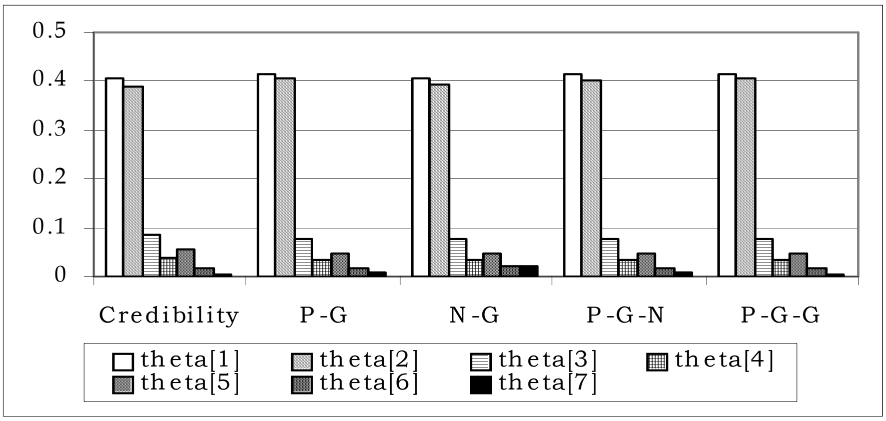
0.09724/(0.09724+0.094+0.01884+0.00781+0.01083+0.00368+0.00172)=***0.4153,***
0.09400/(0.09724+0.094+0.01884+0.00781+0.01083+0.00368+0.00172)=***0.4045***,
....,
0.00172/(0.09724+0.094+0.01884+0.00781+0.01083+0.00368+0.00172)=***0.0073.***
(see the below *italic* figure in the **[Figure 3.1]**).

The other figures from Normal-Gamma, Poisson-Gamma-Normal, Poisson-Gamma-Gamma model are obtained by the same way.

Figure 3.1. The comparison of delay effect

	Credibility model	**P-G**	**N-G**	**P-G-N**	**P-G-G**
theta(1)	0.4070	*0.4153*	0.4045	0.4130	0.4162
theta(2)	0.3894	*0.4045*	0.3937	0.4022	0.4051
theta(3)	0.0857	*0.0787*	0.0768	0.0787	0.0787
theta(4)	0.0396	*0.0333*	0.0343	0.0336	0.0331
theta(5)	0.0561	*0.0462*	0.0463	0.0466	0.0460
theta(6)	0.0164	*0.0157*	0.0225	0.0166	0.0151
theta(7)	0.0057	*0.0073*	0.0219	0.0093	0.0060

Graph 3.1. Comparison among delay effects



3.2 The comparison of results for the calculations

A. Estimation of the total IBNS amount by several models:

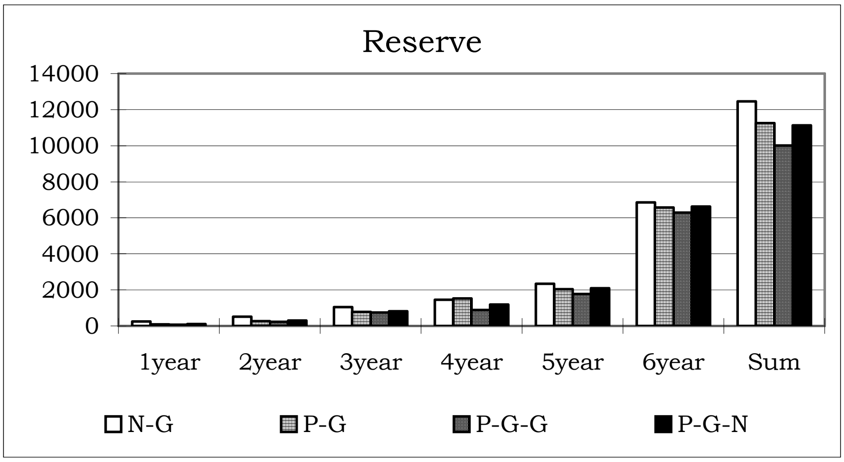
We have four different results based on MCMC methods; Poisson-Gamma, Normal-Gamma, Poisson-Gamma-Normal (via Metropolis-Hastings sampler), Poisson-Gamma-Gamma model. These results give us some useful information about the estimated reserves and the estimation errors. As mentioned, these data are consisted of the claim numbers.

This indicates that the hidden assumption for the distribution of claim number can be fitted well by the Poisson density. These results show that in general Poisson related models have small size of estimated reserves and standard errors in comparison to the credibility model and Normal-Gamma model. The MCMC models show relative small estimated IBNS reserve and standard errors, and the interval of estimated reserves move between 10,722 ~ 12,452 and the interval of standard errors show low 1,193 ~ 2,916 (See **[Figure 3.2], [Graph 3.2]** and **[Graph 3.3]**). All of the estimated reserve and errors by the MCMC models are smaller than that of the credibility model (the estimated reserve 12,146 and standard error 1,239). However, we cannot conclude which model a better candidate is, because these models have various estimated reserves and ratio of standard error to estimated reserves. We discuss the measure again in the next step.

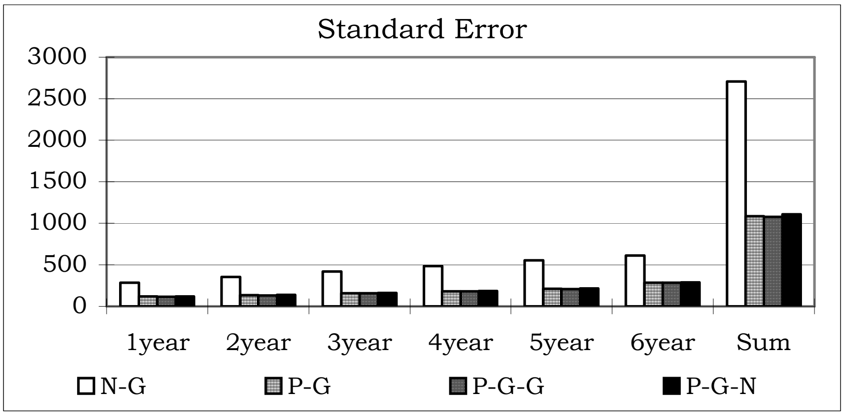
Figure 3.2. Comparison between the estimated reserves and standard errors

	Credibility-model	**N-G**	**P-G**	**P-G-G**	**P-G-N**
Estimated reserves	12,146	12,452	10,891	10,722	11,135
Standard Error	1,239	2,916	1,193	1,180	1,215

Graph 3.2. Yearly reserves and total reserve



Graph 3.3. Yearly standard error and total standard error



B. Definition and calculation of standard error (mean square error):

The results show that the credibility model has the large estimated reserves and standard errors in comparison with MCMC methods. The simulations and comparison between credibility model and several MCMC methods show that the fully Bayesian approach using MCMC methods produces the smaller standard errors than that of credibility model. The Normal-Gamma model has too large size of standard errors in comparison to other models.

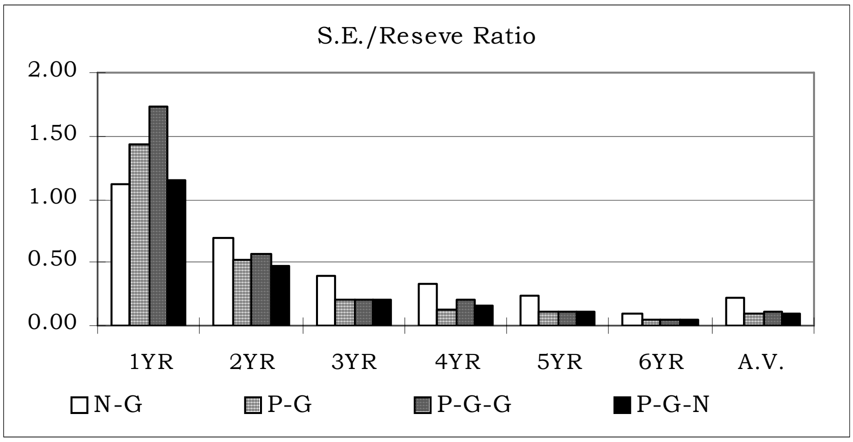
We can choose more practicable and at the same time neither overestimated nor underestimated models, which give small size of the ratio of standard error to estimated reserves (see **[Figure 3.3]** and **[Graph 3.3]**). If we use only the credibility model, the risk of overestimation or underestimation cannot be avoided. Some models of MCMC methods give more information about the estimated reserves and standard errors. In all models, we can choose a better candidate model. The Poisson-Gamma model or Poisson-Gamma-Normal model can be better candidate according to the ratio of standard error to estimated reserves. For the conservative

IBNS strategy, we prefer to accept the Poisson-Gamma-Normal model rather than to accept Poisson-Gamma-Gamma model. It is of interest that the Normal-Gamma model is similar to the credibility model in the estimated reserve.

Figure 2.3. Yearly standard error and total standard error

Years	N-G	P-G	P-G-G	P-G-N
1 year	1.12	1.14	1.73	1.15
2 year	0.69	0.51	0.56	0.47
3 year	0.40	0.20	0.21	0.20
4 year	0.33	0.12	0.20	0.15
5 year	0.24	0.10	0.12	0.10
6 year	0.09	0.04	0.05	0.04
Average	0.22	0.10	0.11	0.10

Graph 3.4. Yearly ratio of standard error to estimated reserves (A.V. means Average)



4. The interpretation of procedure of claim reserve

Loss reserves constitute the largest single item in an insurer's balance sheet and especially in the line of liability business. An upward or downward 5-10% movement of loss reserves could change the whole financial picture of the company. We have argued for the use of stochastic models, especially in assessing the variability or uncertainty inherent in loss reserves. The loss reserve, which is carried in the balance sheet, will be realized in the future. Future paid losses may be regarded as a sample path from the estimated distributions.

The estimated distributions include both statistical risk and estimation risk. The forecast distributions are accurate provided by the assumptions about the future. For example, if it is assumed that future inflation trend has a mean of 10% and a standard error of 2%, and in two years time it turns out that the inflation is 20%, then the forecast distributions are far form accurate. There are three main problem facing a major reserving exercise: Reserving a general class of business of

reasonably homogeneous characteristic; Reserving for a specific event or an individual (usually large) claim; Reserving for a 'phenomenon' (such as, for instance, mass tort claims, asbestos and pollution claims.)
Fundamental to the reserving process is an understanding of the uncertain nature of the technical reserves and the establishment of a framework to monitor progress as claims become paid and as new reserves are established.
The reserving process has the following principal stages.

1. Establish the purpose of the exercise, in the term of reference given, the related department and other likely recipients of the results.
2. Obtain background information and data sets for performing projections. This will include numerical data and objectives of the business.
3. Analyze and check the data to identify any unusual features and reconcile the data to the published accounts or other reference points.
4. Clarify points of detail on the data and, if necessary, obtain more extensive data.
5. Perform projections, possibly using several reserving techniques, i.e. stochastic Chain-ladder method, credibility models, fully Bayesian approach using MCMC methods.
6. Analyze and interpret the projection results and obtain feedback from related department.
7. Finalize the projections and document the calculations and rationale for making specific assumptions, paying particular attention to the more subjective areas.

The wide range of contingencies that can give rise to claims, and the influence that factors beyond the insurer's control (such as taxes, social inflation, legislative charges) have upon claims, means that ultimate level of claims can never be known with certainty until the last claim has been finally settled. The error that reserve estimates are subject to can be thought of in three principal components.

1. Statistical error: Claims might, on average, be distributed by amount and through time according to some well-defined pattern or structure. However, since they are inherently variable, it might possible to express their ultimate level in terms of statistical random error of the underlying distribution.

2. Estimation error: Since policy terms and conditions cannot, in practice, restrict the behavior of claims to predefined distributions, the estimation of the parameters of the distribution is itself subject to estimation error.

3. Model error: In describing the above error components it is implicitly assumed that, through some process of prior information, the general structure of claims development is known (for example, some models assume that incremental payments in respect of a particular underwriting cohort follow a log normal pattern). However there is no guarantee that a selected model is fundamentally the correct one.

Accordingly, any prediction interval computed from the forecast distributions is conditional on the assumptions about the future remaining true. By MCMC methods we can implement easily the inflation effect in the model and moreover we can produce the percentile loaded reserve against such trends (for instance, 105-

110% of the estimated reserve). The uncertainty in loss reserves for the future should be based on a stochastic model that may bear relationship to reserves carried by the company in the past.
The uncertainty for each line for each company should be based on a stochastic model, derived from the company's experience. A model appropriate for one loss development trend will not be appropriate for another.

5. Conclusion

We considered the problem of predicting unpaid losses but not yet reported claims with some credibility models and MCMC methods. The main aim of this thesis is to develop a stochastic macro models. These models are divided into three categories in this thesis. The first models are related with credibility model: variance component model and unconditional credibility model. The second models are cross-classification model with credibility estimator and without credibility estimators. The third models are exact Bayesian model and fully Bayesian model using MCMC methods.

For the IBNR estimation, we can make some useful check points. This check points allow us choose which model is more reasonable, acceptable and predictable:

1. Check that all the assumption contained in the model is satisfied by the data.
2. Calculate the estimated reserves and standard errors of estimates.
3. Easily update models and track forecasts as new data arrive.

The apparent profitability and solvency of a business is highly dependent upon the reserve level and the reserving philosophy. Most of the key financial performance statistics used by insurance company analysts depends in some way upon the reserve level. Reserving is therefore a fundamental aspect of business management. The insights that the reserving process provides into past claim performance and policy exposures can influence the terms and conditions offered on future business and are usually the basis of decisions to cease underwriting certain classes or to withdraw from insurance entirely and support alternative enterprises that are expected to offer better rates of return on capital.

Some models have to satisfy the assumptions, simplicity, acceptability and so on. But in the credibility model we need some assumptions, which can lead sometimes meaningless and artificial by real data. By fully Bayesian approach we can calculate directly the estimates from the data without such assumptions of credibility.

In the meaning of this criterion e.g. simplicity, acceptability, we can compare the credibility models with the fully Bayesian approaches. The credibility models for ratemaking have the same structure as the credibility model for IBNS estimation, which is linear Bayesian estimation. The unknown parameter $\mu(\theta)$ involves the whole structure of estimation in the credibility model. When some assumptions are violated, we can choose some alternative models. The MCMC estimation procedure is developed which efficiently generates the posterior joint of the parameters and regimes. The complete likelihood function is generated at the same time, enabling estimation of posterior probabilities for use in model selection. The procedure can readily be extended to produce joint prediction densities for the variables, incorporating both parameter and model. If the full conditional distribution exists explicitly, the Gibbs sampler produces the posterior distribution. The Gibbs sampler will often be useful where a complicated process can be built up from the components with standard conditional distribution. A particularly useful application of the Metropolis-Hastings sampler is where an intractable density arises within a Gibbs sampler as the product of a standard density and another density, e.g. $\pi(x) \propto \varphi(x) \cdot \phi(x)$, where $\phi(x)$ is a standard density that can be sampled. Fully Bayesian approaches via MCMC method, which are a kind of exact Bayesian estimation using simulation, produce many advantages:

1. Fully Bayesian approaches can be easily inserted and used the information of the premium volume or number of policies.

2. The inflation effect can be inserted and analyzed without any manipulations for the undeflated original data.

3. The models can be selected by the estimated reserves and the ratio of standard error to estimated reserves.

4. If the estimation errors are large, we can use the 105% or 110% percentile loaded loss reserves for conservative reserving policy, which MCMC methods can produce.

The simulations and comparisons between credibility models and several MCMC methods show that the fully Bayesian approach using MCMC method produce a smaller estimation errors and the ratio of standard error to estimated reserves than that of credibility model, which lead good results for the estimated reserves.

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